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A HEURISTIC APPROACH TO THE INDEX TRACKING PROBLEM: A CASE STUDY OF THE TEHRAN EXCHANGE PRICE INDEX

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ABSTRACT

Index tracking, the most popular form of passive fund management, is a portfolio selection problem in which the return of one of the stock market indexes is reproduced by creating a tracking portfolio consisting of a subset of the stocks included in the index. Index tracking has been known as an NP-Hard problem, and sophisticated approaches have been proposed in the literature to solve this problem. This paper presents an easy-to-implement heuristic solution to this complex problem. The proposed approach was implemented to develop a tracking portfolio of 438 stocks listed in the Tehran Exchange Price Index. The numerical results indicate that the approach is able to identify quality solutions within reasonable model runtime.

Keywords: index tracking, portfolio selection, fund management, heuristic approach

INTRODUCTION

Fund management concerns the investigation of stocks and securities of those companies whose stocks are represented in stock markets worldwide (Beasley, Meade, & Chang, 2003). There are two common strategies in fund management. The first is the *active strategy,* which is used when the goal is to make investments in stocks that are expected to outperform other stocks in the market or outperform the average outcomes of the stock market (Alexander & Dimitriu, 2005). In this strategy, performance depends on the fund managers' experience and judgment. The other strategy is called *passive strategy*, which is used by less experienced fund managers who are reluctant to take great risks. In passive strategy, the primary focus is placed on the long-term performance of the market instead of the short-term, temporary extra return achievement (Alexander & Dimitriu, 2005).

Empirical analysis in recent years has revealed that actively managed funds cannot exceed their comparative index, and this has resulted in more attention being paid to passively managed funds in which managers can accrue profits without taking excessive risks. Passive management of funds, particularly index tracking, has gained popularity in the U.S., Europe, Australia and Eastern Asia (European Asset Management Association, 2001; Maringer, 2008). The most common model of passive fund management is called the *index fund* or the *index tracking portfolio*, which takes two forms: *full replication* and *partial replication* (Maringer, 2008). In full replication, the investor purchases all of the stocks available in the index, whereas in partial replication, only a *subset* of stocks are purchased (Shapcott, 1992).

This paper uses partial replication for solving the index-tracking problem, in which choosing a subset of stocks leads to a combinatorial optimisation problem. We have developed an heuristic approach to solving this complex problem. Here, the problems are choosing the best subset of stocks to be included in the index tracking portfolio and calculating the optimal weight for each stock in the tracking portfolio. We tackle the two problems in a disaggregated approach. Although some complicated solutions have been proposed in the literature to solve the index tracking problem, this paper proposes a simple heuristic approach for selecting the optimal subset of stocks, and the concept of pseudo inverse in advanced algebra is used to determine the optimal weight of each stock in the tracking portfolio.

LITERATURE REVIEW

The index tracking problem (ITP) is a portfolio optimisation problem, a popular research area in the field of management science and operations research (MS/OR). Despite the importance of the ITP, only a few published articles address practical solutions to this problem. In fact, there were only 15 published articles that focused on solving the ITP before 2003 (Beasley, Meade, & Chang, 2003). Several approaches and data collections have been used to solve this problem, the comparison and analysis of which are beyond the scope of this paper. Here, we only refer to the most recent and relevant articles in this area.

By establishing the standard mean-variance model for portfolio optimisation, Markowitz created an index tracking portfolio in which the optimal weight of each stock is determined by solving Markowitz's model with its well-recognised objective function and constraints (Derigs & Nickel, 2003; Markowitz, 1952). Hodges (1976) used Markowitz's model and for the first time presented a comparison between the index tradeoff curve and the index tracking portfolio tradeoff curve. Later on, Roll (1992) used Markowitz's model; however, the theoretical deficiency prevented this model from being widely used during the past two decades (Beasley et al., 2003).

After Markowitz, several approaches have been used for modelling the ITP. Rudd (1980) created a single factor model for the ITP of the S&P 500 with an heuristic solution for handling the problem constraints. He included the transaction costs of purchasing and selling stocks in the objective function. Coreilli and Marcellino (2006) presented multi-factor models for solving the ITP. Connor and Leland (1995) considered the cash management problem in their model for building a tracking portfolio and included the transaction costs as a fixed percentage of the money invested. Buckley and Korn (1998) modified Connor's model and considered transaction costs as the main model constraint. Rudolf, Wolter and Zimmermann (1999) developed four different forms of linear functions for index tracking models. They changed the tracking error minimisation model to a linear model but could only solve the problem for a small set of data. Frino and Gallagher (2001) studied the impact of seasonal factors on tracking errors and found that the tracking error is higher in January because of the market fluctuations at the beginning of the year.

Because of the complexity of the ITP, traditional methods cannot solve the problem optimally; therefore, heuristic and meta-heuristic approaches have recently been adopted to identify the optimal solutions to the problem (Beasley et al., 2003; Derigs & Nickel, 2003; Krink, Mittnik, & Paterlini, 2009; Maringer & Oyewumi, 2007; Miao, 2007; Ruiz-Torrubiano & Suárez, 2009). Beasley et al. (2003) used an evolutionary heuristic approach for identifying the best stocks to be placed in the portfolio. Derigs and Nickel (2003) adopted a simulated annealing meta-heuristic to design a decision support system to be used by fund managers. Okay and Akman (2003) used a constraint aggregation method for the first time to solve the ITP; this method was initially developed by Beasley et al. (2003). They demonstrated that their approach yields similar computational results within a shorter model runtime. Oh, Kim and Min (2005) used a Genetic Algorithm (GA) approach to follow the South Korean stock market index. Dose and Cincotti (2005) developed a clustering model to build a portfolio to track the S&P 500 index. In this model, the stocks are grouped based on a distance measure between two stocks, and only one single stock is chosen as a representative of that group. By solving a quadratic problem, the optimal weight for each is stock is calculated.

Krink et al. (2009) suggested a differential, evolution-based meta-heuristic and compared the results with those of other meta-heuristic algorithms such as GA, simulated annealing and particle swarm optimisation. It was shown that this approach is more efficient in solving more complicated problems. Maringer and Oyewumi (2007) used an identical approach to re-create the Dow Jones index performance and demonstrated several advantages in terms of the number of parameters involved and also its ease of implementation. Primbs and Sung (2008) used stochastic receding horizon control for solving an ITP. They developed a predictive control model and formulated the ITP as a stochastic linear quadratic control problem. Carakgoz and Beasley (2009) used a linear regression to solve the problem in which both the ITP and improved indexation problem are transformed into a mixed integrated linear program that can be solved by standard linear solvers such as IBM ILOG CPLEX Optimization Studio. Barro and Canestrelli (2009) studied a dynamic ITP that considers the minimum number of stocks in an index portfolio in terms of transaction costs. They proposed a multi-stage tracking error model that was solved using a stochastic planning technique based on a progressive hedging algorithm. Ruiz-Torrubiano and Suárez (2009) introduced a hybrid strategy based on the combination of an evolutionary algorithm and quadratic programming. They used the formulation of Beasley et al. (2003) and demonstrated that the computational times can be reduced using their hybrid approach.

In this paper, we propose an heuristic approach for solving a complex ITP. Our heuristic approach helps the investor determine both the optimal subset of stocks for the tracking portfolio and the optimal weight of each stock. To investigate the effectiveness of the solution proposed in this paper, the approach is implemented to develop the desired tracking portfolio from 438 stocks presented in the TEPI.

ITP FORMULATION

We assume having N stocks in a given index during time period T. The objective is to develop an index tracking portfolio with K stocks (in which $K << N$) to track the index from the time of T to the future time of L. The answers to the following queries are sought in a typical ITP: (1) which stocks must be included in the tracking portfolio—i.e., the optimal K stocks from the set of N stocks; and (2) what proportion of the invested fund should be allocated to each of the K stocks. A basic approach for modelling an ITP is an historical approach that assumes that the past directs the future. Like many previous works, such as those of Beasley et al. (2003) and Maringer and Oyewumi (2007), we investigated the validity of this assumption by dividing the data set into in-sample data and out-of-sample data. Therefore, the problem formulation presented in this section is similar to the formulations of Beasley et al. (2003), Maringer and Oyewumi (2007), Ruiz-Torrubiano and Suárez (2009) and Okay and Akman (2003).

Mathematical Modelling

We use the following parameters for the ITP formulation.

N : Total number of stocks in the concerned stock market index
K : Desired number of stocks in the tracking portfolio
e : Minimum proportion of investment in each stock
x^u : Maximum proportion of investment in each stock
T : Time period (0, 1, 2,…, T) in which the time unit can be disaggregated to days or weeks
$S_{t,i}$: Value of one unit of stock i (i = 1, 2, …, N) at time t (t = 0,…, T)
$r_{t,\mathrm{I}}$: Single period continuous time return of stock i at time t (derived from Equation 1).

$$r_{t,i} = Log_e \left(S_{t,i} / S_{t-1,i} \right) \tag{1}$$

B_o : Amount of investment to create the index tracking portfolio (i.e., the available budget)
n_i : Number of stocks i (I = 1, . . . , N) in the index tracking portfolio (decision variable)
b_i : Stock selection binary variable; $b_i = 1$ if stock i is selected to be included in the tracking portfolio, $b_i = 0$ otherwise (decision variable)
P_t : Value of index tracking portfolio in time t (derived from Equation 2):

$$\sum_{i=1}^{N} n_i S_{t,i} = P_t \tag{2}$$

$r_{t,P}$: Single period continuous time return of the tracking portfolio at time t (derived from Equation 3):

$$r_{t,p} = Log_e \left(P_t / P_{t-1} \right) \tag{3}$$

I_t : Value of the index at time t
$r_{t,I}$: Single period continuous time return of the concerned index at time t (derived from Equation 4):

$$r_{t,I} = Log_e \left(I_t / I_{t-1} \right) \tag{4}$$

Using the above parameters, Equation 5 formulates the objective function for the proposed ITP.

$$Min\,(TE) = Min\left(\sum_{t=1}^{T}\left(r_{t,P} - r_{t,I}\right)^{2}/T\right)^{1/2} \tag{5}$$

Subject to:

$$\sum_{i=1}^{N} b_i \leq K \qquad i = 1\ldots N \tag{6}$$

$$x^{l} b_i \leq (n_i S_{t,i} / B_o) \leq x^{u} b_i \qquad i = 1\ldots.N \tag{7}$$

$$\sum_{i=1}^{N} n_i S_{T,i} \leq B_o \qquad i = 1\ldots N \tag{8}$$

$$b_i \in [0,1] \qquad i = 1\ldots.N \tag{9}$$

$$n_i = 0, 1, 2,3,\ldots, N \qquad i = 1\ldots.N \tag{10}$$

Equation 5 represents the objective function of the proposed ITP. The mean square error ratio is used to accommodate the theoretical weakness in using variance measure. In addition, the objective function does not intend to achieve excess return of the concerned index, but to track it as closely as possible. Equation 6 is the main model constraint that ensures there are K stocks available in the tracking portfolio. Equation 7 ensures that if stock i is not available in the index portfolio ($b_i = 0$), then n_i will be equal to zero; and if stock i is available in the index portfolio ($b_i = 1$), then the minimum and maximum investment limits on each stock must be considered in the solution approach. The complete use of the available budget is imposed by Equation 8. Beasley et al. (2003), Ruiz-Torrubiano and Suárez (2009) and Krink et al. (2009) argue that it is not necessary to consider this constraint in an ITP because a small investment in some stocks or budget reduction can simply rectify small deviations for the complete satisfaction of this constraint. Equations 9 and 10 enforce restrictions on the integer variables.

Modified Mathematical Modelling using Continuous Decision Variables

There are two integer variables in the proposed formulation: (1) b_i is a binary variable indicating whether stock i is selected in the tracking portfolio, and (2) n_i is an integer variable standing for the number of stocks in the portfolio. Obviously, n_i is a discrete number because the number of purchased stocks cannot be a decimal or a rational number. To tackle the complexity of solving a discrete model, we convert n_i to a continuous form as is also proposed in Derigs and Nickel (2003), Frino, Gallagher and Oetomo (2005), and Ruiz-Torrubiano and Suárez (2009). We define x_i as a continuous variable to show the proportion of investment in stock *i* derived from Equation 11.

$$x_i = \frac{n_{i,t} \cdot S_{i,t}}{B_0} \qquad i = 1 \ldots N \tag{11}$$

Using this approach, n_i can fluctuate dynamically over time, although the proportion of the investment in stock *i* remains constant. Thus, $r_{t,p}$ can be reformulated as follows:

$$r_{t,p} = \sum_{i=1}^{N} r_{t,i} x_i \tag{12}$$

With this modification, the formulation of the objective function is changed to the following:

$$Min\,(TE) = Min\,\frac{1}{T}\left(\sum_{t=1}^{T}\left(\sum_{i=1}^{N} r_{t,i} x_i - r_{t,I}\right)^2\right)^{1/2} \tag{13}$$

If stock *i* exists in the tracking portfolio, then b_i = 1 and constraints 14–16 are effective:

$$x^L \le x_i \le x^U \tag{14}$$

$$\sum_{i=1}^{N} x_i \le 1 \tag{15}$$

$$\sum_{i=1}^{N} b_i \le K \tag{16}$$

If stock *i* does not exist in the tracking portfolio, then b_i = 0 and thus x_i = 0.

$$x_i \in R \tag{17}$$

$$b_i \in [0,1] \qquad i = 1 \ldots N \tag{18}$$

In this formulation, Constraint 14 is replaced with Constraint 7, indicating the floor and the ceiling proportions of the amount of money invested in stock *i*.

Constraint 15 is replaced with Constraint 8, ensuring the complete use of the available budget. Constraint 16 is similar to Constraint 6, which is the major constraint of the ITP, restricting the number of stocks in the tracking portfolio. Short selling is allowed in our model, and therefore Equation 17 indicates that x_i can also obtain negative numbers.

A HEURISTIC SOLUTION APPROACH

Defining continuous variables in previous section can significantly simplify the development of a solution approach for the proposed ITP. However, a binary variable b_i remains in the model formulation. The problem of selecting the stocks to be included in the tracking portfolio can significantly increase the problem dimension, which in many cases results in an extremely large problem search space (Rudolf et al., 1999). This is why heuristic and meta-heuristic methods have been adopted in recent years. In general, two questions must be answered to build a tracking portfolio: (1) which K stocks of N stocks should be selected from the concerned index for inclusion in the tracking portfolio, and (2) what is the optimal weight of each selected stock in the tracking portfolio. Assuming that the answer to the first question is known to be S comprised of K stocks and also assuming that all model constraints are relaxed, the problem can be solved using Equation 19.

$$Min\left(TE\right) = Min\,\frac{1}{T}\left(\sum\nolimits_{t=1}^{T}\left(\sum_{i\in S} r_{t,i}x_i - r_{t,I}\right)^2\right)^{1/2} \tag{19}$$

For better illustration of this formulation, we define the following matrixes to convert Equation 19 into a matrix form. $A_{T\times N}$ is the return matrix of N stocks in the time horizon T, and $X_{N\times 1}$ is the decision variable matrix of the optimal weights of stocks in the tracking portfolio. $I_{T\times 1}$ is also defined as a matrix for the return of index in time horizon T. Therefore, the matrix form of Equation 19 is presented in Equation 20.

$$Min\left(TE\right) = Min\left(\sum \left(AX - I\right)^2\right) \tag{20}$$

For Equation 20 to be solved, Equation 21 must be true because *TE* is minimised (i.e., equal to zero) only if the return of the index-tracking portfolio (*AX*) equals the return of the index (*I*).

$$AX = I \tag{21}$$

Although matrix *A* is a non-square matrix, Equation 21 can be calculated using the pseudo inverse technique from the context of advanced linear algebra (Hefferon, 2008). Hence, matrix *X* can be calculated from Equation 22.

$$X = \left(A^T A\right)^{-1} A^T I \tag{22}$$

In summary, by relaxing all the model constraints, the optimal objective function can be calculated from Equation 22. However, the feasibility of the solution is not guaranteed if model constraints are taken into consideration. In addition, the second question of the ITP has nevertheless remained unanswered. In any ITP, a time series of data (i.e., index time series) is tracked by the use of another time series of data (portfolio time series). Similarity between the concept of "the correlation between two time series of data" and the ITP leads us to development of an heuristic approach in which the selected stock to be included in the portfolio is the one with the strongest positive correlation with the index. In the proposed approach, the correlation between the time series of the value of *N* stocks (included in the index) and the time series of the value of the index is calculated. The first *K* + *L* stocks with the highest positive correlation are selected to form the reduced search space, and the remaining *N*-*K*-*L* stocks are ignored. *L* is an integer between 0 and 10 aiming to make the search space a little wider for achieving the optimal tracking portfolio (it will be demonstrated in the next section why the value of *L* should be in this range). Using this approach will help considerably in reducing the size of the search space, making the problem easier to solve optimally. Hence, the proposed approach presented in this section can be summarised in the following four steps:

1. Select *K* + *L* stocks out of *N* stocks that have the highest positive correlation with the index. This approach reduces the problem search space in which the number of candidates in the tracking portfolio is *K* out of *K* + *L* ($\binom{K+L}{K}$) instead of *K* out of *N* ($\binom{N}{K}$).
2. From Equation 22, calculate the optimal weight of the stocks in every selected portfolio.

3. From Equation 20, calculate the *TE* value for each tracking portfolio.

4. Select the portfolio with minimum *TE* to represent the index.

Although the complex ITP can easily be handled using this approach, there are nevertheless uncertainties in terms of the feasibility of the generated solutions because of the ignored model constraints. To deal with this, we define a constraint violation measure representing the deviations against the major model constraints. If the value of the constraint violation measure is large and significant, then our heuristic approach must be adopted to take the model constraints into consideration. If the constraint violation measure is not substantial, we ignore the consideration of model constraints because the proposed approach has previously resulted in a feasible solution. The detailed discussion of the implementation of this model on the TEPI is presented in the next section.

MODEL IMPLIMENTATION

To evaluate the performance of the proposed heuristic approach in this paper, we implemented our heuristic method to develop the tracking portfolio in a real-life case study. The most demanding stock market in Iran is the Tehran Stock Exchange (TSE), the price index of which is referred to as the TEPI (also known as TEPIX). The TSE is made up of 438 stocks providing a representation of the country's economy throughout the year. For our data set, we decided to use the daily stock prices of the TEPI in 2003 because in that year the country experienced the most stable economy of the past decade and in 2003 there was much less missing data, enabling us to create a quality data set. Similar to the methods of Maringer and Oyewumi (2007), the average of the existing prices of the adjacent days was used for the missing data. Having 205 values for each stock in TEPI, we clustered the time period [0, 103] for the in-sample data set and the time period [103, 205] for the out-of-sample data set.

The proposed model was run on a SONY VAIO laptop with a 2.1 GHz dual-core processor and 2GB of RAM. In addition, we used $x^l = 0.01$ and $x^u = 1$ as the minimum and maximum proportions of investment in each stock, respectively. Table 1 shows the achieved numerical results for the first TEPI tracking portfolio for $K = 5$ and $K = 10$. As previously mentioned, the main issue in the presented heuristic approach is to identify whether the generated tracking portfolio is feasible. For this, we measured the percentage of constraint violation for both floor and ceiling constraints as well as the budget constraint (i.e., the ratio of the violated portfolios to all possible portfolios in the problem search space).

Columns 5 and 6 in Table 1 (constraint violation) demonstrate that these values are quite small in scale, which validates the feasibility of the solutions found.

To evaluate the effectiveness of the generated search space obtained from our heuristic approach, the mean and standard deviations of tracking error (TE) from all possible portfolios in the search space are calculated in columns 7 and 8, respectively. The low values of "standard deviation of TE" and the proximity of the values of "mean of TE" and their equivalent in column 2 (i.e., the minimum TE) not only demonstrate the effectiveness of this approach in generating a practicable search space but can also validate the fundamental concept behind the proposed approach in this paper (i.e., the correlation-based selection for the tracking portfolio—refer to previous section for more information).

Table 1
Computational results for the TEPI for K = 5 and K = 10

	Tracking error (TE) and Time (Second)			Constraint violation		Search space effectiveness	
1	8	7	6	5	4	3	2
K	Standard deviation of TE	Mean of TE	Budget	Floor and Ceiling	Time	Out-of -sample	In-sample
2.43147e-05	2.61463e-04	0	1.66500e-03	1.52	7.59397e-04	2.03214e-04	5
1.94316e-05	2.17834e-04	0	5.41254e-05	20.37	5.95365e-04	1.65864e-04	10

The promising results demonstrate that the approach could develop an effective tracking portfolio in terms of both tracking error and model runtime. The TE values in Table 1 show only a small difference between in-sample data and out-of-sample data. Moreover, the results for $K = 10$ are slightly better than those for $K = 5$, which indicates that TE decreases with an increase in the number of stocks in the tracking portfolio.

The achieved numerical results indicate that the index-tracking problem with an historical look-back approach (refer to Beasley et al., 2003) can be effectively addressed using the concept of the matrix correlation (which is naturally quite similar to the concept of index tracking). In this manner, the complicated index-tracking problem can be solved easier and faster (refer to column 4 in Table 1 for the runtime report). A feasible tracking portfolio consisting of five stocks could be produced in about two seconds. Figure 1 and Figure 2 illustrate the performance of our tracking portfolio with $K = 5$ for in-sample and out-of-sample data, respectively. The achieved results for $K = 10$ are shown in Figure 3 and Figure 4.

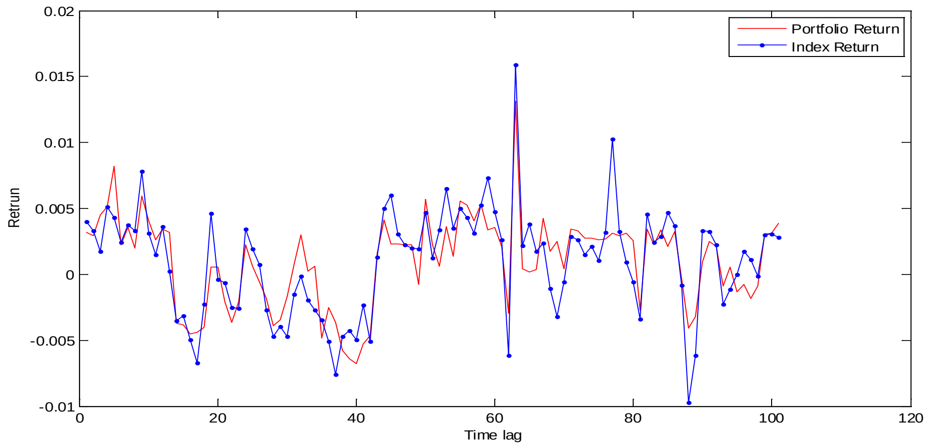

Figure 1. *In-sample tracking performance for TEPIX with K = 5.*

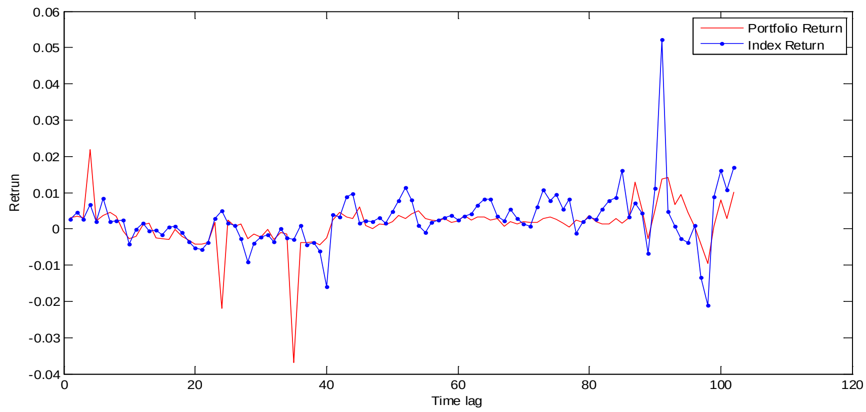

Figure 2. *Out-of-sample tracking performance for TEPIX with K = 5.*

In the former section, we mentioned that the value of L should be within the range of 0 to 10 as an indication of the size of the search space. To evaluate this assumption, we run our heuristic model for both $K = 5$ and $K = 10$ when the value of L varies between 0 and 10. In Figures 5 and 6, the vertical axis demonstrates the *TE* values and the horizontal axis represents $K + L$, in which L changes from 0 to 10. Both figures show an initial sharp decrease in the value of tracking error (TE) while $K + L$ increases to approximately 11 (i.e., $L = 6$) and 18 (i.e., $L = 8$), respectively. The *TE* value seems to remain almost unchanged after these points. This indicates that the suggested movement range for L is well set to be between 0 and 10, resulting in an appropriate search space for our heuristic.

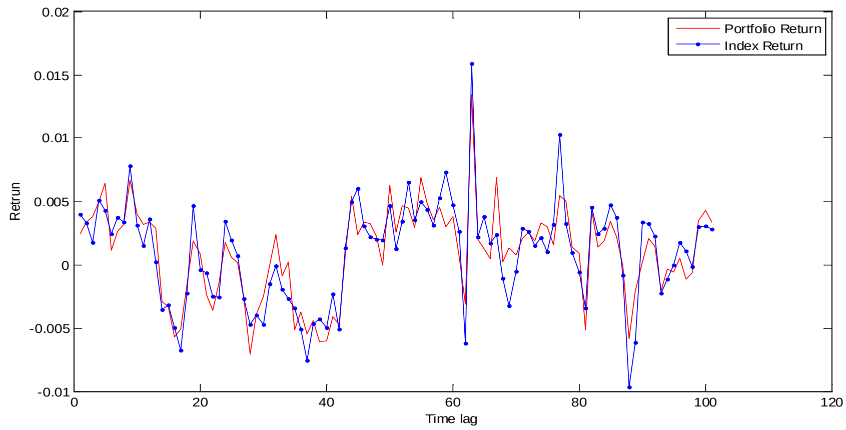

Figure 3. *In-sample tracking performance for TEPIX with K = 10.*

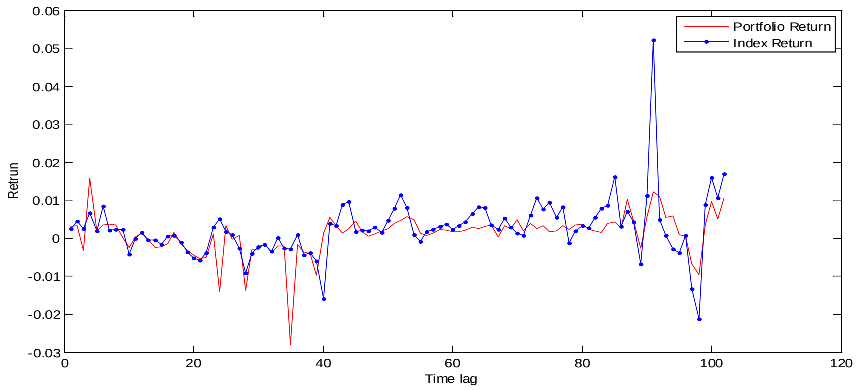

Figure 4. *Out-of-sample tracking performance for TEPIX with K = 10.*

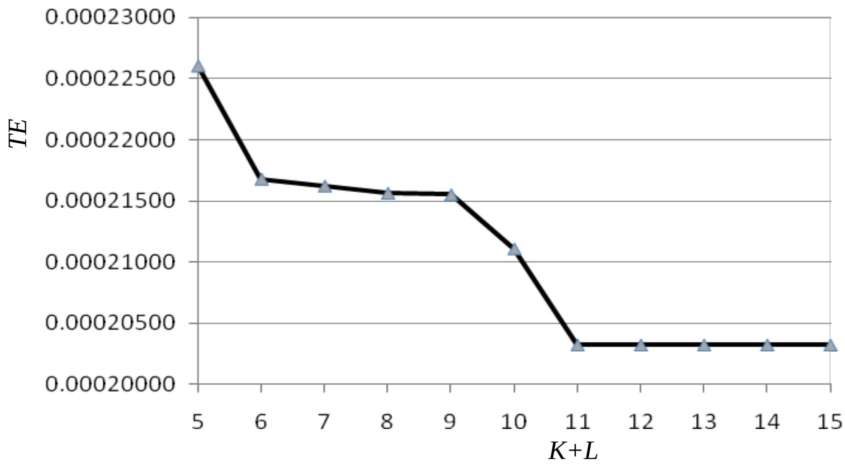

Figure 5. Changes in *TE* value with fluctuations in the value of *L* (*K* = 5)

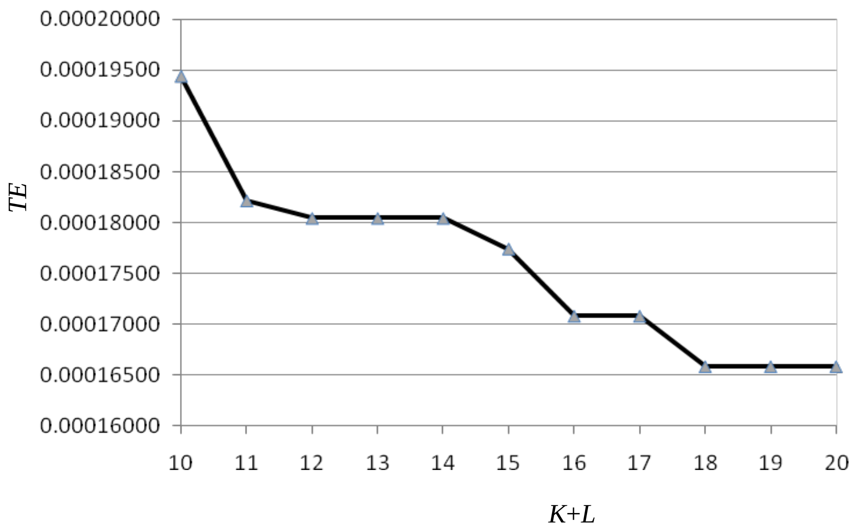

Figure 6. Changes in *TE* value with fluctuations in the value of *L* (*K* = 10)

CONCLUSIONS

The index tracking problem is commonly faced by fund managers who intend to develop tracking portfolios for following or out-performing the average stock market performance. The available heuristic and meta-heuristic techniques proposed for solving index tracking problems are complex in nature and require long computational times. This paper presented an easy-to-implement heuristic approach using a correlation-based method to select the appropriate stocks for inclusion in the tracking portfolio as well as the concept of pseudo inverse to determine the optimal weight of the selected stocks.

The proposed approach was implemented to develop a tracking portfolio from 438 stocks listed in the Tehran Exchange Price Index. The numerical results indicate that our heuristic approach yields quality outcomes with small tracking error values in both in-sample and out-of-sample data sets within reasonable model runtime. With this proven application, the proposed method can easily be implemented in other stock markets to assist fund managers in dealing with the complexity of the index-tracking problem.

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