

STRUCTURAL CHANGE ANALYSIS OF ACTIVE CRYPTOCURRENCY MARKET

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ABSTRACT

Motivated by the large frequent price fluctuation and excessive volatility observed in the cryptocurrency market, this study adopts Bai and Perron's structural change model by incorporating the trading volume and autoregressive variables to examine the number and location of change points in daily closing price, return and volatility proxied by the squared return of Cryptocurrency Index, Cryptocurrency Index 30, and the top 10 cryptocurrencies ranked according to market capitalisation. Results show that the structural changes occur very frequently for the price series, followed by squared return and return series which were consistently observed between December 2017 to April 2018. In addition, the results also reveal that the two cryptocurrency indices may not be beneficial as an indicator to reflect the whole cryptocurrency market for the entire studied period as these two indices do not display consistent structural change in contrast to the top 10 cryptocurrencies that might have significant implications for modelling the cryptocurrency data.

Keywords: Structural change point, cryptocurrency, index, return, volatility

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INTRODUCTION

The world had witnessed an explosive growth in the cryptocurrency market in the year 2017 mainly for Bitcoin (BTC) which recorded a considerable price appreciation of approximately 1300% (www.coinmarketcap.com). One of the interesting aspects of the cryptocurrency market is the exceptional large price and volatility fluctuations observed in a short span of time. News had been continuously reporting prices of BTC and altcoins which reached an all-time high, especially in late 2017 when there was an abrupt surge of interest in the market. This can be noted from some sudden spikes in the total of cryptocurrency market capitalisation and trading volume in terms of the U.S. dollar as presented in Figure 1. A large retracement soon occurred that caused the total market capitalisation to reduce by about 50% from the peak recorded in January 2018. Investors are undoubtedly exposed to additional risks in a volatile market, and according to Williams (2014), the volatility of BTC is relatively large as compared to other traditional financial assets such as precious metals, Standard and Poor 500 and the U.S. dollar.

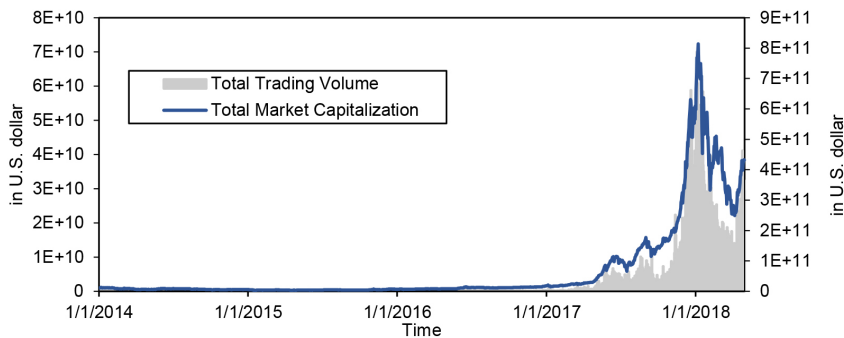


Figure 1. Total market capitalisation and total trading volume for cryptocurrency market from January 2014 to April 2018.

Source: www.coinmarketcap.com

Different models have been employed in the foregoing studies to estimate the volatility of cryptocurrencies. Katsiampa (2017), for instance, fitted the BTC data to six types of generalised autoregressive conditional heteroskedasticity (GARCH) models, in which the autoregressive-component GARCH was found to be the best model to estimate volatility. There were seven cryptocurrencies considered in Chu et al. (2017): BTC, Dash, Litecoin (LTC), MaiSafecoin, Monero, Dogecoin and Ripple (XRP) which are ranked according to market capitalisation. They had fitted 12 different GARCH models to each cryptocurrency and the results showed that the Glosten-Jagannathan-Runkle GARCH (GJRGARCH) with normal innovation fitted well for Dogecoin, the GARCH model with normal innovation for XRP, and integrated GARCH (IGARCH) model with normal innovation for others. Lahmiri et al. (2018) examined the long-range memory in the volatility series of

seven BTC markets via fractionally IGARCH (FIGARCH). Philip et al. (2018) incorporated the stylised facts displayed by cryptocurrencies such as generalised long memory effect, leverage and heavy tails to the stochastic volatility model and there were 224 different cryptocurrencies being applied to determine which of these properties exist. Peng et al. (2018) used GARCH, exponential GARCH and GJR-GARCH models with three different innovations: normal, Student-t, and skewed Student-t distributions to contrast the nine models with the support vector regression (SVR)-GARCH (SVRGARCH) model. They had come to a conclusion that the estimated mean using the volatility equations from SVR, SVRGARCH outperforms the other nine GARCH models. Instead of estimating the volatility as a latent process, some authors estimate the volatility directly using range-based measures. Among them, Tan et al. (2020) estimated the volatilities of 102 active cryptocurrencies using the Garman and Klass (1980) range-based volatility measures and model the resulting volatilities using the asymmetric bilinear conditional autoregressive range model by incorporating leverage effect. Wu et al. (2020) used the Parkinson range-based volatility measure (Parkinson, 1980) to estimate the volatility of BTC and model the volatility using the component conditional autoregressive range model to capture the long memory property of the BTC volatility. One issue that arises from these studies is that most of the studies only focus on one single model that can best fit the entire period of volatility which may not be adequate to capture the abrupt shift observed in the cryptocurrency market when the presence of structural break is not taken into consideration. Bauwens et al. (2015) tested the presence of structural breaks in 23 macroeconomic series such as gross domestic product and consumer price index and demonstrated the importance of addressing the presence of structural change in forecasting. They are in the view that, in the presence of structural change, there is no single model that can always provide the optimal result in forecasting.

Numerous studies on cryptocurrencies have attempted to segment return series, based on different criteria, into several independent partitions in order to investigate the volatility of data. Urquhart (2016) highlighted that the BTC market is inefficient if the sample is in the full length of data from 1 August 2010 to 31 July 2016. However, if the data were separated into two sub-samples by imposing a split after the dramatic surge in BTC price in late 2013, it was found that the market inefficiency of BTC was largely attributed to the first sub-sample while the second sub-sample showed an improvement in BTC market efficiency. Bouoiyour and Selmi (2016), however, noticed that despite the remarkable BTC price appreciation over time, its volatility rate remained at a less pronounced rate since January 2015. Hence, they estimated the volatility of BTC over two main periods: before January 2015 and after January 2015 by employing different models for the two sample periods. To verify the break of BTC data around the 2013 price crash,

Bouri et al. (2016) tested the structural break in BTC price around the period before modelling the volatility of BTC. They then separated the long data into two segments: before the 2013 price crash and after the 2013 market price. Thies and Mólnar (2018) investigated the presence of change points in the BTC return series using the Bayesian structural change model. Forty-eight change points on the average return of BTC were detected and the segments were combined that exhibited the same properties into 7 regimes based on their volatility. Bouri et al. (2019) determined structural change of logarithmic BTC price, absolute return and squared return series via Bai and Perron (2003) approach where four and five change points were respectively observed in these series derived from the two different exchange platforms, Bitstamp and Coindesk, in which, a maximum allowable of change in each series is only five. Their findings also encompassed change points of price cash in December 2013. Mensi et al. (2019), on the other hand, revealed the impact of structural change on the volatility of BTC and Ethereum (ETH), and suggested that ignoring the presence of structural changes may lead to the persistency of overestimation in volatility. Nevertheless, there has been thus far relatively little progress on the determination of the number and location of change points in price, return and volatility proxied by squared return for cryptocurrency indices, and altcoins in which this research gap should appropriately be filled.

Our aims in this paper are threefold. Firstly, as many previous studies tend to focus only on BTC or on a limited number of popular cryptocurrencies, we provide statistical analysis for the top ten cryptocurrencies ranked according to market capitalisation as of 30 April 2018, namely, BTC, ETH, XRP, Bitcoin Cash (BCH), EOS, Cardano (ADA), LTC, Stellar (XLM), IOTA and Tron (TRX). In addition, this study is also extended to the whole cryptocurrency market by advocating the use of the cryptocurrency index as an indicator to typify the entire market, in which, the two cryptocurrency indices considered are Cryptocurrency Index (CRIX) and Cryptocurrency Index 30 (CCI30). Price, return and squared return series are the primary inputs in risk management and portfolio selection since the price will reflect the market information while the return and squared return will represent the expected gain and volatility in financial modelling and financial applications (see Tsay, 2010; Hossain & Ismail, 2021; Tan et al., 2022). As for the factor that influences the dynamics of cryptocurrency data, daily trading volume is often reported to be the significant factor that influences the BTC price level (Kristoufek, 2015; Urquhart, 2018). Secondly, we will apply the technique introduced by Bai and Perron (2003) to detect the number and location of change points in price, return and squared return of the series for the cryptocurrencies and indices by incorporating the significant autoregressive terms, and exogenous variables such as daily trading volume in the structural change model. Finally,

it is of paramount importance to examine the performance of the two indices in representing the cryptocurrency market as a whole. To this end, we would compare and evaluate the consistency of the findings by collating the individual cryptocurrency results with the other two indices.

DATA

CRIX and CCI30

Two cryptocurrency indices, CRIX and CCI30 daily closing prices in U.S. dollars, are retrieved from their official websites: <http://crix.hu-berlin.de> and <https://cci30.com>. CRIX data spans from 8 August 2014 to 30 April 2018 consisting of 1,362 observations introduced by Trimborn and Härdle (2016). The number of constituents in the index is chosen in steps of five based on a lengthy time-varying selection method that relies on Akaike Information Criterion. More specifically, the number of constituents is not fixed but is recalculated quarterly based on AIC measure to ensure the up-to-date fit to the current market situation (see Trimborn & Härdle, 2016). CCI30 data on the other hand starts on 9 January 2015 and ends on 30 April 2018 with a total of 1208 observations which were acquired from Rivin and Scevola (2018). The index members are chosen to be the first 30 cryptocurrencies ranked according to market capitalisation. The base number for CRIX and CCI30 are 1,000 and 100, respectively. Both indices are rebalanced every month and reconstituted for each quarter.

Return series are obtained by taking the difference of two consecutive log prices and squared return series are computed by taking the square of return which is commonly used as an indicator for variance or risk in financial terminology (see Tan et al., 2019). Figure 2 illustrates the price, return and squared return series for CRIX and CCI30. The vertical dotted lines are the estimated change points while the red lines represent the fitted equation estimated using Equation (1) in the presence of change points which is discussed in Empirical Analysis section. The large fluctuation of the cryptocurrency market can be observed by the strikingly sharp increase in price since the year 2017 which is soon followed by an abrupt drop in the year 2018. This corresponds to the spikes noticed in the return and squared return series indicating by high volatility level for the cryptocurrency market. As seen in Figure 2, the fitted equations (in red colour) are able to capture the prominent spikes observed in the series.

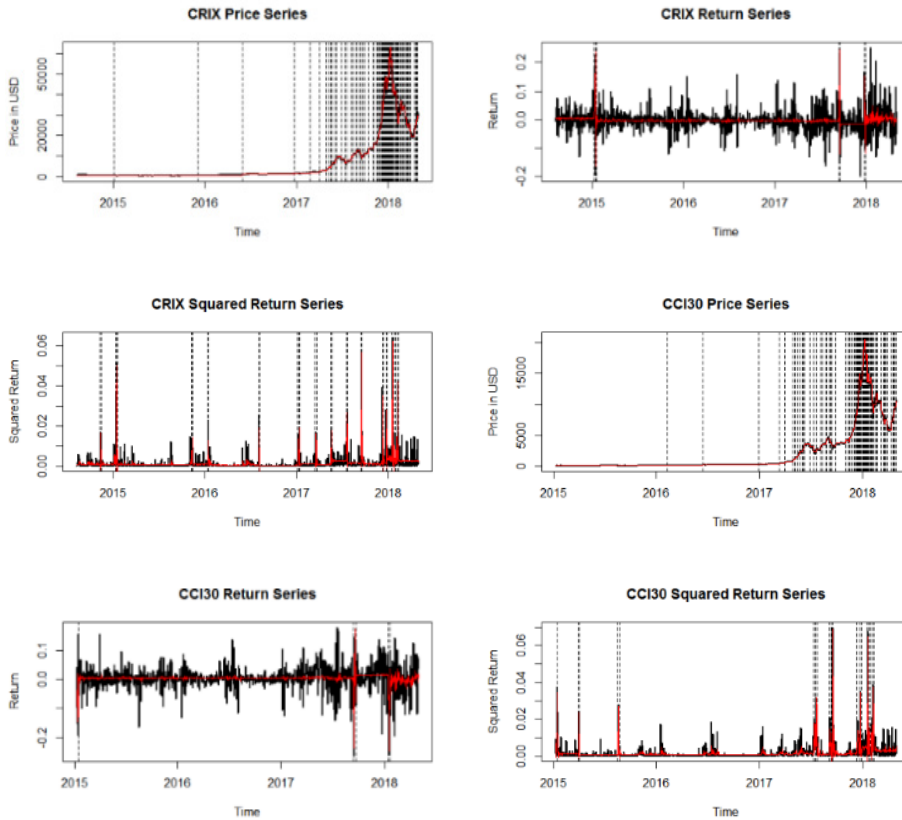


Figure 2. Price, return and squared return series for CRIX and CCI30 (Data obtained from <http://crix.hu-berlin.de> and <https://cci30.com>)

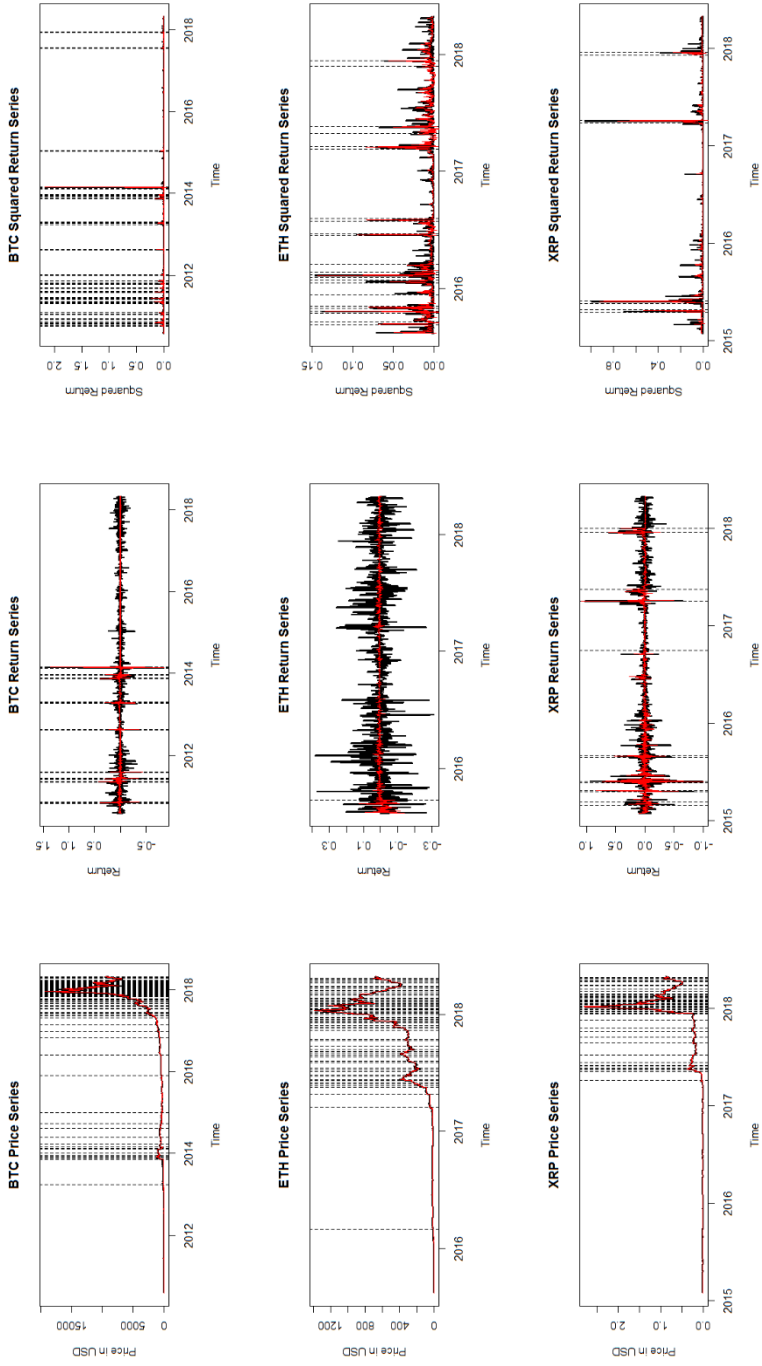
Top 10 Cryptocurrencies

Daily closing prices in U.S. dollars of the top 10 cryptocurrencies were retrieved from Yahoo Finance (<https://finance.yahoo.com>). These cryptocurrencies constitute 79% of the total market capitalisation with BTC dominating 37.04% of the overall market as of 30 April 2018. This is BTC that has been introduced since the year 2010 whereas most cryptocurrency data are introduced in year 2017 onward. Table 1 presents the summary statistics of the indices and daily closing prices for these 10 cryptocurrencies. From Table 1, it was observed that more than 50% of times the observed closing prices of cryptocurrencies will earn below their expected return in which all these indices and cryptocurrencies display positively skewed and leptokurtic distributions.

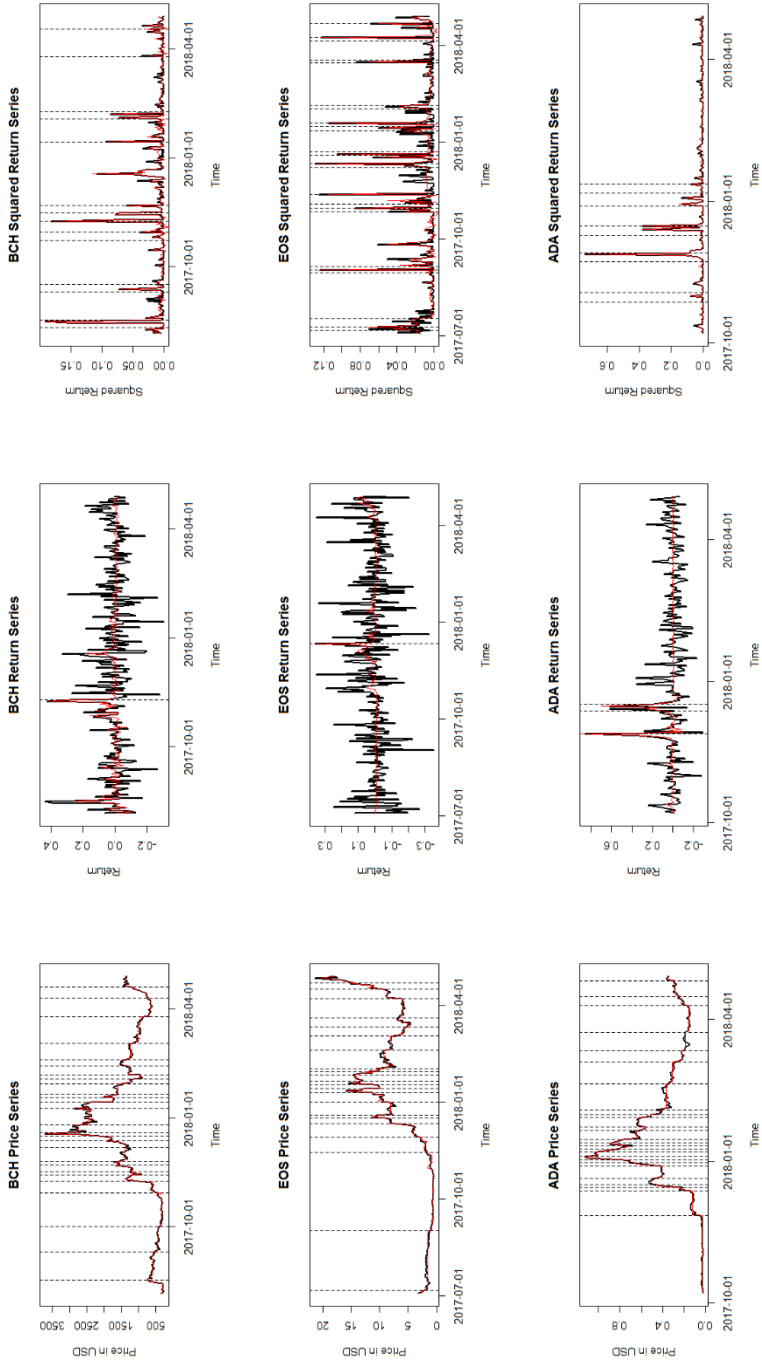
Table 1
Summary statistics of the indices and top ten cryptocurrencies' closing prices

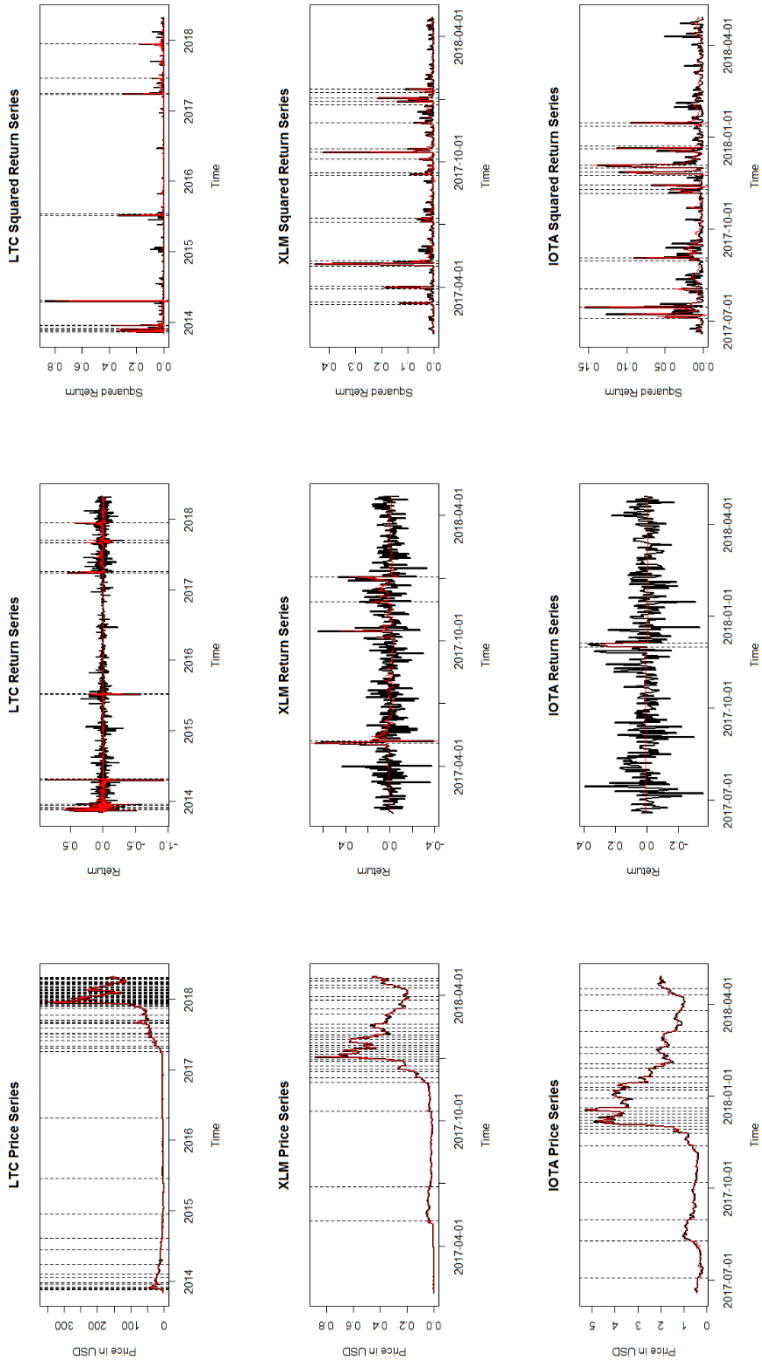
Name	Data starts from	Count	Min	Median	Mean	Max	Excess kurtosis	Skewness
CRIX	8 August 2014	1,362	342.07	1,082.93	6,193.32	62,895.26	5.9828	2.5084
CCI30	9 January 2015	1,209	57.46	265.94	2,062.60	20,800.95	5.6367	2.3983
BTC	1 August 2010	2,830	0.06	263.07	1,139.02	19,345.49	13.6895	3.6092
ETH	13 August 2015	992	0.42	12.83	179.84	1,385.02	2.8428	1.8441
XRP	28 January 2015	1,190	0.00	0.01	0.17	2.78	14.9229	3.5231
BCH	7 August 2017	267	274.48	1,015.19	1,108.36	3,715.91	0.6760	1.0216
EOS	5 July 2017	300	0.49	3.63	5.09	21.41	0.0882	0.8792
ADA	7 October 2017	206	0.02	0.22	0.28	1.13	1.1138	1.1812
LTC	31 October 2013	1,644	1.12	4.10	28.28	357.51	8.8267	2.9662
XLM	24 January 2017	463	0.00	0.03	0.13	0.88	1.4624	1.5256
IOTA	19 June 2017	316	0.16	0.97	1.40	5.32	0.6188	1.2413
TRX	6 October 2017	207	0.00	0.04	0.04	0.30	11.3857	2.6603

Figure 3 illustrates the price, return and squared return series for ten cryptocurrencies: BTC, ETH, XRP, BCH, EOS, ADA, LTC, XLM, IOTA and TRX. The vertical dotted lines represent the estimated change points while the red line is the fitted equation estimated using Equation (1) in the presence of change points. Price series appear to have the most change points compared to the return and squared return series. As indicated in Figure 3, the fitted equations (in red colour) are able to capture the changes of the series, especially during prominent spikes. The details of change point detection are discussed in Empirical Analysis section.



Structural Change Analysis of Active Cryptocurrency Market





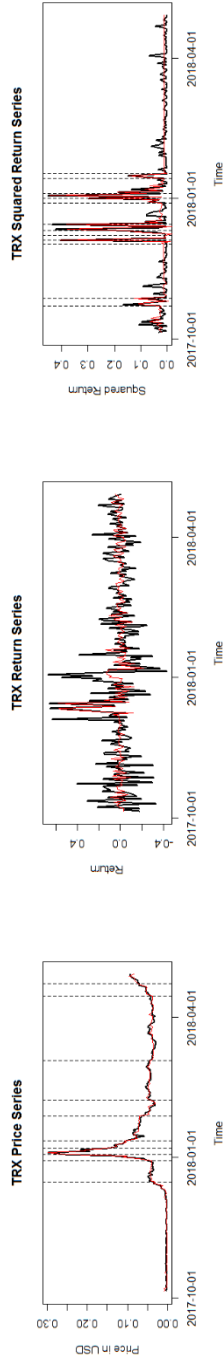


Figure 3. Price, return and squared return series for the top 10 cryptocurrencies.
Source: Data from <https://finance.yahoo.com>

METHODOLOGY

The multiple change point modelled by Bai and Perron (2003) is applied to estimate the number and location of change points in price, return and squared return series of the two cryptocurrency indices and the top 10 cryptocurrencies by incorporating the significant exogenous and autoregressive variables to the model. The exogenous variable refers to the trading volume whereas the autoregressive variables are the lagged values of the respective return series and squared return series.

To test the significance of these variables, we regress the three time series data, namely price, return and squared return on a constant, trading volume and autoregressive variables. The constants, trading volume and autoregressive variables are employed to study which variables can explain the return series and squared return series of cryptocurrencies, while for the case of price series, only the constant and trading volume (except for CRIX and CCI30) are considered excluding the autoregressive variables, as price series are assumed to be following a random walk.

Table 2 presents the parameter estimates of trading volume and autoregressive terms on price, return and squared return series for different types of cryptocurrencies. The findings reveal that the trading volume indeed has a significant impact on price series for all cryptocurrencies except for ADA. With regards to the return series, trading volume is a significant exogenous variable for XRP, BCH, EOS, ADA, LTC and XLM, and for the squared return series, the trading volume is a significant exogenous variable for all cryptocurrencies except for BTC, XRP and TRX. It is noticed that trading volume has an impact merely on the BTC price series but not for the return and squared series. This result can further be substantiated by Balcilar et al. (2017) who had shown that the trading volume can be used to predict BTC return only when the market is not in the extreme mode such as in its bearish or bullish condition. Moreover, they also highlighted that the volatility of BTC is not appropriate for prediction purposes through trading volume. After the significant autoregressive variables and exogenous variables for the price, return and squared return series are identified, these significant variables are then incorporated into the multiple structural change model to detect the change points.

Table 2
Parameter estimates and standard errors (in parentheses) for price, return and squared return series of various indices and cryptocurrencies

Price series												
Cryptocurrency	CRIX	CCI30	BTC	ETH	XRP	BCH	EOS	ADA	LTC	XLM	IOTA	TRX
<i>h_t</i> size	0.0015	0.0018	0.0015	0.0035	0.003	0.012	0.012	0.010	0.002	0.008	0.010	0.020
Constant	6193.3*** (307.9)	2070.2*** (107.1)	398.3*** (23.610)	66.93*** (6.035)	0.0959*** (0.0071)	838.5*** (41.50)	3.100*** (0.2096)	0.2807*** (0.0018)	18.450*** (1.0910)	0.0624*** (0.0075)	0.9524*** (0.0537)	0.0182*** (0.0022)
Trading Volume	-	-	5.79e-06*** (5.40e-08)	9.81e-07*** (2.35e-08)	2.35e-09*** (5.58e-11)	2.29e-06*** (1.99e-07)	4.37e-08*** (2.39e-09)	5.87e-11 (4.29e-10)	3.12e-07*** (9.17e-09)	1.63e-08*** (9.60e-10)	1.30e-08*** (7.29e-10)	1.07e-08*** (6.07e-10)
Return series												
Cryptocurrency	CRIX	CCI30	BTC	ETH	XRP	BCH	EOS	ADA	LTC	XLM	IOTA	TRX
<i>h_t</i> Size	0.0025	0.0025	0.0022	0.005	0.005	0.017	0.010	0.020	0.004	0.008	0.0065	0.025
Constant	-0.0022* (0.0011)	0.0031* (0.0012)	0.0040*** (0.0013)	0.0072*** (0.0027)	0.0016 (0.0033)	-0.0127 (7.30e-03)	-0.0050 (0.0069)	-7.24e-04 (0.0093)	4.96e-05 (0.0020)	-0.0026 (0.0063)	0.0046 (0.0066)	0.0037 (0.0134)
Trading Volume	-	-	-4.40e-12 (3.01e-12)	-1.08e-11 (1.04e-11)	8.50e-11** (2.64e-11)	1.62e-10*** (3.85e-11)	2.49e-10** (8.52e-11)	1.12e-09*** (2.57e-10)	7.63e-11*** (1.74e-11)	3.46e-09*** (8.88e-10)	-2.55e-11 (9.54e-11)	4.69e-09 (3.92e-09)
Lag 1	-0.0141 (0.0270)	-0.0065 (0.0286)	0.0462* (0.0188)	-0.0637* (0.0315)	-0.3281*** (0.0291)	0.0461 (0.0617)	-0.0410 (0.0589)	-0.1376 (0.0732)	-0.1280*** (0.0247)	0.0026 (0.0048)	0.0591 (0.0573)	-0.0022 (0.0707)
Lag 2	-0.0160 (0.0270)	0.0200 (0.0286)	-0.1696*** (0.0188)	0.0060 (0.0313)	0.0629* (0.0304)	-0.1400* (0.0620)	0.0106 (0.0581)	0.1669* (0.0680)	-0.0542* (0.0247)	-0.0064 (0.0467)	-0.0354 (0.0574)	0.1772* (0.0709)
Lag 3	0.0400 (0.0270)	0.0640* (0.0287)	0.0225 (0.0191)	0.0707* (0.0313)	-0.0211 (0.0304)	-0.0084 (0.0602)	0.0398 (0.0572)	0.0325 (0.0713)	0.0160 (0.0247)	-0.0027 (0.0463)	0.1107 (0.0578)	0.2051** (0.0709)
Lag 4	0.0095 (0.0271)	-0.0069 (0.0287)	0.1070*** (0.0190)	-0.0151 (0.0312)	0.0191 (0.0304)	-0.1122 (0.0586)	-0.0576 (0.0494)	-0.0159 (0.0693)	0.0407 (0.0248)	-0.0778 (0.0456)	0.0057 (0.0572)	-0.1390* (0.0702)
Lag 5	0.0127 (0.0272)	0.0190 (0.0286)	0.0835*** (0.0190)	0.0013 (0.0312)	-0.0435 (0.0304)	0.0201 (0.0575)	-0.0238 (0.0496)	-0.0929 (0.0681)	0.0032 (0.0247)	0.0788 (0.0463)	0.0188 (0.0555)	0.0179 (0.0697)
Lag 6	0.1055*** (0.0272)	0.1205*** (0.0286)	-0.0092 (0.0190)	-0.0009 (0.0293)	0.0492 (0.0289)	0.0154 (0.0571)	0.0602 (0.0493)	-0.0645 (0.0682)	0.0870*** (0.0246)	0.0152 (0.0459)	0.0992 (0.0553)	-0.0370 (0.0695)

(Continued on next page)

Table 2 (Continued)

Squared return series												
Cryptocurrency	CRIX	CCI30	BTC	ETH	XRP	BCH	EOS	ADA	LTC	XLM	IOTA	TRX
<i>h_t</i> size	0.003 (0.0001)	0.005 (0.0015)	0.002 (0.0008)	0.008 (0.0005)	0.008 (0.0018)	0.025 (0.0017)	0.012 (0.0016)	0.030 (0.0045)	0.004 (0.0009)	0.010 (0.0021)	0.010 (0.0016)	0.015 (0.0062)
Constant	0.0008*** (0.0001)	0.0008*** (0.0015)	0.0021* (0.0008)	0.0029*** (0.0005)	0.0057** (0.0018)	0.0022 (0.0017)	0.0650*** (0.0016)	0.0112* (0.0045)	0.0032*** (0.0009)	0.0044* (0.0021)	0.0055*** (0.0016)	0.0112 (0.0062)
Trading Volume	-	-	6.19e-13 (1.96e-12)	3.50e-12* (1.75e-12)	1.76e-11 (1.38e-11)	6.61e-11*** (8.31e-12)	9.24e-11*** (1.64e-11)	9.64e-10*** (1.27e-10)	1.55e-11* (7.33e-12)	1.26e-09*** (2.69e-10)	9.08e-11*** (1.98e-11)	2.70e-09 (1.55e-09)
Lag 1	0.2657*** (0.0272)	0.1821*** (0.0284)	0.0279 (0.0186)	0.2042*** (0.0317)	0.4496*** (0.0291)	0.1804* (0.0582)	-0.0648 (0.0570)	-0.1591* (0.0733)	0.5157*** (0.0248)	0.3408*** (0.0469)	0.1065 (0.0565)	0.0396 (0.0709)
Lag 2	0.0154 (0.0281)	0.0373 (0.0287)	0.1406*** (0.0186)	0.0379 (0.0309)	-0.0275 (0.0318)	-0.2546*** (0.0594)	-0.0562 (0.0559)	0.1658** (0.0636)	-0.2092*** (0.0278)	0.0596 (0.0489)	-0.0262 (0.0568)	0.2049** (0.0710)
Lag 3	0.0001 (0.0281)	0.0613* (0.0286)	0.0087 (0.0183)	0.0747* (0.0308)	0.1375*** (0.0318)	0.0258 (0.0580)	0.0003 (0.0056)	-0.1959*** (0.0671)	0.0911** (0.0283)	0.0064 (0.0488)	0.0837 (0.0565)	0.1204 (0.0719)
Lag 4	0.1012*** (0.0279)	0.1124*** (0.0286)	0.2144*** (0.0183)	0.0642* (0.0307)	-0.0519 (0.0318)	-0.0413 (0.0515)	0.0117 (0.0189)	-0.0191 (0.0647)	0.0085 (0.0283)	-0.0620 (0.0487)	0.0003 (0.0560)	0.0789 (0.0714)
Lag 5	0.0489 (0.0281)	0.0974*** (0.0287)	-0.0070 (0.0186)	0.0519 (0.0300)	0.0827** (0.0318)	-0.0046 (0.0512)	0.0244 (0.0180)	-0.0699 (0.0633)	0.0019 (0.0278)	0.0877 (0.0488)	-0.0098 (0.0518)	-0.0766 (0.0700)
Lag 6	0.0118 (0.0282)	0.0375 (0.0283)	0.1664*** (0.0186)	-0.0305* (0.0143)	-0.0238 (0.0291)	0.0950* (0.0477)	0.0116 (0.0178)	-0.1064 (0.0638)	0.0223 (0.0247)	-0.0779 (0.0455)	0.0332 (0.0511)	0.0354 (0.0700)

Notes: *significance at the 5% level; **significance at the 1% level; ***significance at the 0.1% level.

Liu et al. (1997) who first considered multiple structural change using the least-squares method partitioned the data into m segments in order to compute the sum of squared residuals for each segment with the idea that the change point estimators used are regarded as the global minimisers of the sum of squared residuals. Bai and Perron (2003) extended their work by applying a dynamic programming algorithm to estimate the global minimisers of the sum of squared residuals. The algorithm uses at most least-squares operations of order $O(T^2)$ for any number of m change points which appears to be a more efficient way of achieving a minimum global sum of squared residuals with T representing the length of data.

Consider a linear model below with m changes ($m+1$ segments):

$$y_t = x_t' \boldsymbol{\beta} + z_t' \boldsymbol{\delta}_j + u_t \quad t = T_{j-1} + 1, T_{j-1} + 2, \dots, T_j,$$

for $j = 1, 2, \dots, m + 1$. In this model, y_t is the observed dependent variable at time t with dimension 1×1 ; x_t' is a $1 \times k$ vector of exogenous variables with k number of constant coefficients in vector $\boldsymbol{\beta}$ of dimension $k \times 1$; z_t' is a $1 \times n$ vector of exogenous variables with n number of corresponding coefficients that are subject to change in vector $\boldsymbol{\delta}_j$ of dimension $n \times 1$ and u_t is the disturbance at time t with dimension 1×1 . The indices (T_1, T_2, \dots, T_m) are the estimated change points treated as unknown with $T_0 = 0$ and $T_{m+1} = T$. In this study, we concentrate on pure structural change model which allows all the coefficients to be subject to change by letting $\boldsymbol{\beta} = \mathbf{0}$ so that the shifts of all exogenous variables, if any, are considered. The above multiple linear regression system can then be expressed in its matrix form as below, with $\bar{\mathbf{Z}}$ is a diagonal matrix that partition \mathbf{Z}_j at (T_1, T_2, \dots, T_m) :

$$\mathbf{Y} = \bar{\mathbf{Z}} \boldsymbol{\delta} + \mathbf{U}, \tag{1}$$

or

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{pmatrix} = \begin{pmatrix} \mathbf{Z}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{Z}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{Z}_{m+1} \end{pmatrix} \begin{pmatrix} \boldsymbol{\delta}_1 \\ \boldsymbol{\delta}_2 \\ \vdots \\ \boldsymbol{\delta}_{m+1} \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_T \end{pmatrix}, \quad \text{with } \mathbf{Z}_j = \begin{pmatrix} z_{T_{j-1}+1}' \\ z_{T_{j-1}+1}' \\ \vdots \\ z_{T_j}' \end{pmatrix}$$

where the dimension for \mathbf{Y} is $T \times 1$, for $\bar{\mathbf{Z}}$ is $T \times n(m+1)$, for $\boldsymbol{\delta}$ is $n(m+1) \times 1$ and for \mathbf{U} is $T \times 1$.

For each m -partition (T_1, T_2, \dots, T_m) denoted by $\{T_j\}$, the estimates of $\boldsymbol{\delta}_j$ are evaluated by minimising the sum of squared residuals. Substituting the values

into the objective function and denoting the resulting sum of squared residuals as $S_T(T_1, T_2, \dots, T_m)$, the estimated change points $(\hat{T}_1, \hat{T}_2, \dots, \hat{T}_m)$ are determined by $(\hat{T}_1, \hat{T}_2, \dots, \hat{T}_m) = \operatorname{argmin}_{T_1, \dots, T_m} S_T(T_1, T_2, \dots, T_m)$, where the minimisation is taken over all partitions (T_1, T_2, \dots, T_m) . Consequently, the change point estimators are the global minimisers of the objective function. The global sum of squared residuals for any m -partition (T_1, T_2, \dots, T_m) and for any value of m , must necessarily be a particular linear combination of these $T(T+1)/2$ sums of squared residuals. The estimates of the change points, m -partitions $(\hat{T}_1, \hat{T}_2, \dots, \hat{T}_m)$, will correspond to this linear combination with a minimal value. The dynamic programming algorithm is regarded as a more efficient approach to contrast all possible combinations (corresponding to different m -partitions) in order to achieve a minimum global sum of squared residuals. The number of changes is controlled by the trimming error, h , where h is the ratio of number of days in a segment over the total number of days (count). In this study, we let h to be the smallest possible value so that we would be able to detect the presence of change points even if the changes occur in a short span of time without limiting the number of changes. For estimation purposes, the number of days in a segment must be greater than the number of regressors in the model. It is worth noting that, Bouri et al. (2019) would only allow a maximum of five changes in the detection of structural change process. The above algorithms are implemented using the R package `strucchange` with function `breakpoints`.

EMPIRICAL ANALYSIS

Price Series

Price series is greatly affected by the force of supply and demand, and other external events. It reacts sensitively to news and information in the market. When demand is greater than supply, the price will ascend, and the market is bullish or vice versa. It is noticed that there were few consistent and apparent change points that occurred in the individual cryptocurrencies over time. Figure 4 depicts the monthly segmentation of the price series, in which a change point is represented by an alternate change in the colour of the horizontal bar.

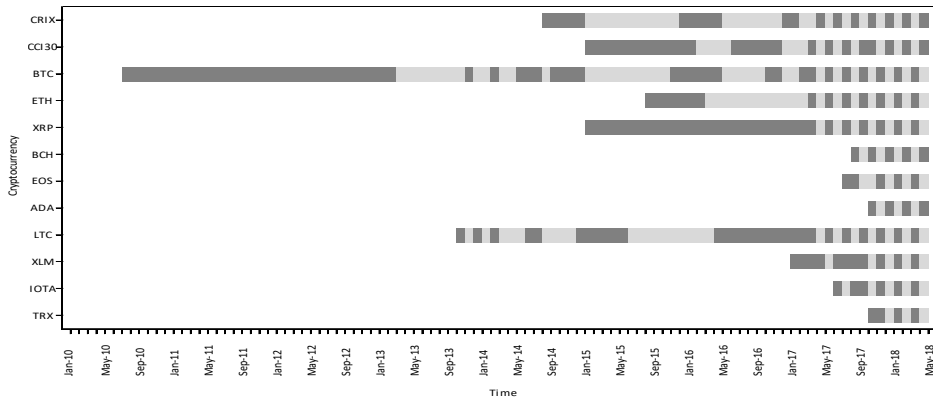


Figure 4. Locations of change point detected in price series

Results reveal that change points in price series are detected specifically at the turning of the year. By observing the longer data such as CRIX, CCI30, BTC and LTC, the changes were detected almost at every ending or beginning of the year commencing from the year 2013 when the cryptocurrency market begins to gain popularity. We hence postulate that the cryptocurrency market is subject to a “year-end” effect as there are cyclical changes in price at the turnings of years. Our results also provide evidence and justification in terms of the location of the change point as indicated in the study conducted by Bouri et al. (2016) which happened during the BTC price crash of December 2013. It is also confirmed that the cryptocurrency market indeed underwent unexpected high price fluctuations in 2017 supported by the presence of change points in approximately each month of 2017 for all data. The detailed number of the detected change points in the respective months throughout the whole sampled period for the price series can be provided upon request.

Return Series

Return is an important variable in finance as it measures the profit of the investment. High return will usually be accompanied by high risk, hence, in the event of high volatility, high return is also anticipated. Figure 5 reports the monthly segmentation of the return series where a change point is represented by an alternate change in the colour of the horizontal bar.

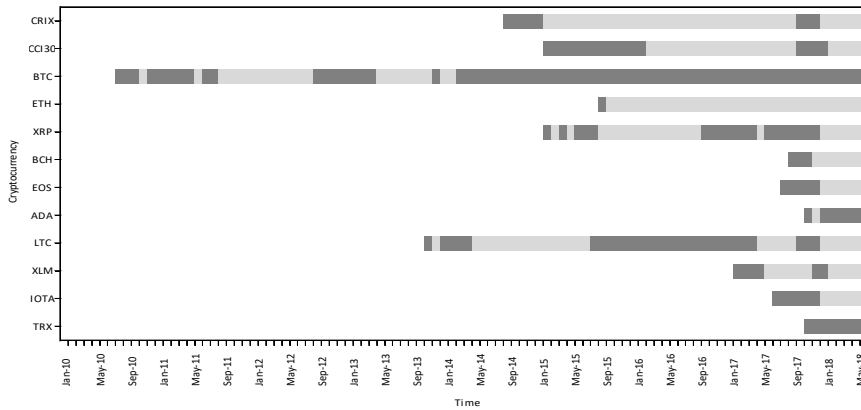


Figure 5. Locations of change point detected in return series

From Figure 5, there was no change point in the BTC return series after the second quarter of the year 2014. We also notice that the ETH return series was detected with only one change point located in September 2015 and with no change point thereafter. On the other hand, there was no change point detected for TRX in its return series.

Among all the change points identified, CRIX and CCI30 consistently indicated the change points in January 2015 and September 2017. These estimated change points may be attributed to some unforeseen events affecting some groups of cryptocurrencies which were unobservable. Moreover, this may also be due to several members of the indices that were reconstituted at every quarter and the ten individual cryptocurrencies might not be the components of the indices at those particular periods of time. Besides, the CRIX return series was also detected with one change point in December 2017 while the change point of the CCI30 return series appeared in late January 2018 during the abrupt market price depreciation. Both these change points were to be expected since the cryptocurrency market was in the process of the change of its trend towards the beginning of years 2017 and 2018 as most of the altcoins also displayed the analogous patterns in this period. The number of detected change points in the corresponding months throughout the whole sampled period for the return series is also made available upon request.

Squared Return Series

Squared return is commonly used to assess the uncertainty of the financial market. In this study, we also approximate the volatility by using the squared return estimates and Figure 6 illustrates the monthly segmentation of the squared return series in which each change point is represented by an alternate change in colour of the horizontal bar.

As can be seen in Figure 6, there are more change points detected in squared return series as compared to return series, especially for ETH where there is only one change point detected in return series, and yet, there were 23 change points in total for squared return series throughout the period (see Table 3). BTC appears to experience more change points in the squared return series in 2011 and tends to become lesser from the year 2012 to the year 2017 with no change point detected in the year 2016. Our finding is in line with the selection of the change point at beginning of the year 2015 as indicated by Bouoiyour and Selmi (2016) that further confirms the volatility change of BTC at that period. Meanwhile, five change points were detected for ETH squared return series whereas most altcoins showed the presence of change points in July 2017, the last quarter of the year 2017 and January 2018. There were also change points detected in the squared return series for both indices which showed consistent signs of change in January 2015, July 2017, and in the turning points of years 2017 or 2018. The number of detected change points in the respective months throughout the whole sampled period for the corresponding squared return series is also made available upon request.

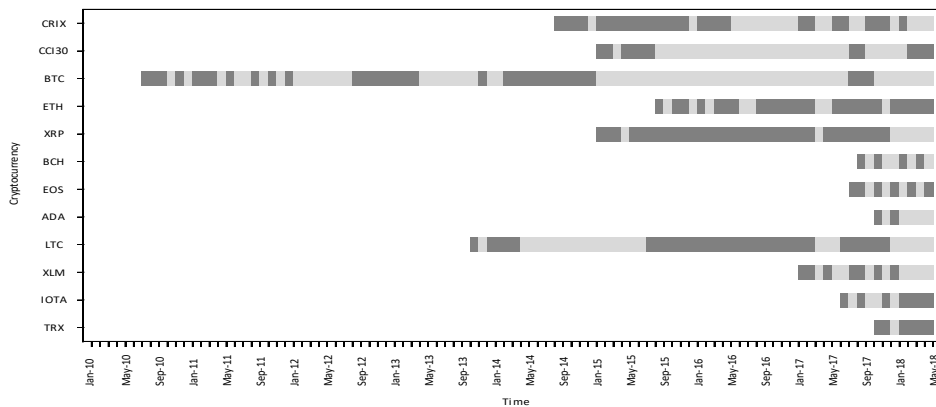


Figure 6. Locations of change point detected in squared return series.

DISCUSSION

As indicated in the results, there were large numbers of change points detected in the price, return and squared return series over the period and it is observed that the detected change points are not consistent among the three series with a greater number of change points detected in price series, followed by squared return series and the least in return series. Table 3 summarises the total number of change points detected according to the types of series and cryptocurrencies.

Table 3
Summary of total number of change points detected in price, return and squared return series

Cryptocurrency	CRIX	CCI30	BTC	ETH	XRP	BCH	EOS	ADA	LTC	XLM	IOTA	TRX
Price	72	68	72	62	45	27	22	26	66	31	27	10
Return	7	6	22	1	13	1	1	3	14	4	2	0
Squared return	31	19	56	23	8	14	24	9	14	20	16	12

It is also noticed that the number of change points detected in CRIX and CCI30 are not consistent with the number of change points identified in the top ten individual cryptocurrencies. Although CRIX and CCI30 comprise a moderate number of selected cryptocurrencies, the constituents are re-evaluated and re-selected each quarter, and hence, the entry of the indices is not the same throughout the period. ElBahrawy et al. (2017) measured the average rank occupation time of cryptocurrencies based on their market capitalisation, and they seemed to warrant two conclusions that the turnover rate of cryptocurrencies is high and the cryptocurrencies keep changing their position in ranks according to market capitalisation. Consequently, the difference in change points detected between the two indices and the individual cryptocurrencies in this study may be due to the high mobility of cryptocurrencies, in which, the 10 individual cryptocurrencies might not be the constituents of CRIX and CCI30 at every point of time throughout the sampled period. Because of its fast-changing position nature, we are in the view that the cryptocurrency index may act as a benchmark for the market at a certain point of time only and may not be optimal to be used as an indicator to represent the entire market for a full length of the sampled period.

The primary issues of the abrupt change in the cryptocurrency market in the turning of the year 2017 or 2018 discussed earlier might be due to the large correction of sharp price appreciation, stricter regulations and government involvement, rumours and negative news, and other technological difficulties. These are the influential factors that may contribute to the instability of the market that requires investors and financial practitioners to be more cautious in the cryptocurrency market.

CONCLUSION

Our empirical findings signify the existence of change points in price, return and squared return series of cryptocurrency data and most of them are not consistent among the three series with most change points detected in the price series and the least change points detected in the return series. Moreover, the locations for

data segmentation carried out in some previous related literatures seem to be in line with our estimated change points. Hence the results may indicate to financial practitioners or researchers to realise the possible instability of parameters that may exist in all aspects of cryptocurrency analysis and modelling process due to the frequent existence of change points affected by underlying internal or external factors.

With the growing interest in the cryptocurrency market, one aspect of our concern is whether the total trading volume is a crucial factor that will influence the price, return and squared return series. In this case, we are in the view that the volume does have a significant impact on the three series, however, different cryptocurrencies show different levels of significance. As to our study, only the BTC price series tends to be significant but not for the return and squared return series.

Additionally, we can see that even though the two cryptocurrency indices, CRIX and CCI30, both consist of a moderate number of cryptocurrencies based on their respective selection methods, the indices do not well-capture the properties of the whole market for the entire sampled period. The locations of change points detected are not consistently close to the change points detected in the top ten cryptocurrencies which may merely imply that the indices are not optimal to represent the cryptocurrency market for those periods partly due to its fast-changing nature.

On the basis of the results, integrating the change point method into financial modelling appears to be beneficial in the prediction process, specifically when one who wants to model the return or volatility in cryptocurrencies. These findings lend support to the wide applications in financial modelling to accommodate in response to the respective change points. Despite the encouraging results of this study as to the positive effect of points detected, perhaps an important area for future research in the years to come will be in the refinement of approaches to the analysis of threshold model, structural change model or regime-switching model.

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