

DETERMINANTS OF IMPLIED VOLATILITY FUNCTION ON THE NIFTY INDEX OPTIONS MARKET: EVIDENCE FROM INDIA

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ABSTRACT

In this paper, we examine two important propositions for the Indian options market: (1) the relationship between implied volatility and moneyness referred to as volatility smile and (2) the potential determinants of the smile asymmetry. We use daily data for the S&P CNX Nifty index call and put options and the underlying market index for the calendar years 2004 and 2005. We find that the volatility functions exhibit a positive slope in the Indian context using alternative measures of moneyness, thus confirming the consistency of our findings. Our evidence on smile asymmetry is in contrast with findings for mature markets, which exhibit negative asymmetry profiles in general. This may be owing to differences in investors' behaviour and market microstructure between mature and emerging markets. We also show that historical volatility and time to expiration are the potential determinants of smile asymmetry in India, as is the case with international evidence. We feel that a strong theoretical foundation should be provided for this observable empirical phenomenon.

Keywords: volatility, moneyness, smile, asymmetry, causality

INTRODUCTION

The global derivatives market, which deals with a number of innovative products introduced over the last few decades, continues to grow at a rapid rate. According to the Futures Industry Association (FIA) report, the total number of futures and options traded on world exchanges reached 11.859 billion contracts in the year 2006. This is a tremendous number and the growth rate is accelerating remarkably. During 2006, total contracts trading volume was 19% higher than in 2005, whereas the total for 2005 was 12% higher than in 2004. In 2005, KOSPI

200 options (Korea Exchange) stood at first rank among global options and futures exchanges with 2593 million contracts, and National Stock Exchange of India (NSE) stood at 14th rank with 131.65 million contracts.

The Securities Exchange Board of India (SEBI) approved trading of derivative contracts on the NSE and Bombay Stock Exchange (BSE) during the year 2000. The derivatives segment of NSE started trading of futures contracts on the Nifty Index on June 12, 2000. After this, trading was introduced on Index and Equity options in June and July of 2001, while interest rate futures began trading in March 2003. Futures and options on other NSE indices, such as CNX IT Index and Bank Index, started trading in August 2003 and June 2005, respectively. The total exchange traded contracts volume of the Indian derivatives market increased by about 88.14% and stood at USD1,236,989.5 million during 2005–2006, where NSE alone accounted for about 99.9% of total traded contracts.

The groundbreaking work of Black and Scholes (1973) on option pricing assumes that all option prices on the same underlying asset with the same expiration date and different exercise prices should have the same implied volatility (IV) (Canina & Figlewski, 1993). However, few studies find that IV tends to differ across moneyness and time to expiration. This is well inferred in the US market on stock indices after the October 1987 crash (Rubinstein, 1994; Jackwerth & Rubinstein, 1996).

The pattern of IVs across time to expiration is usually referred to as the term structure of IVs, whereas the 'skew' or the 'smile' refers to the pattern across moneyness levels. However, in the following the expression 'smile' is used interchangeably with 'skew' and will include the pattern across both time and moneyness, the so-called volatility surface.

The reason for the expression 'smile' is the empirical findings for S&P 500 options before the 1987 stock market crash. The IVs for deep in- and out-of-the-money options were found to be higher than the at-the-money options, thus creating a smile-shaped pattern. After the crash, the smile in S&P 500 options changed to look more like a 'sneer' with monotonically decreasing IV for increasing exercise prices, but the IV pattern is often referred to as the smile regardless of its actual shape. The actual shape of the smile differs between different markets, underlying assets and time periods. For instance, Beber (2001) finds a smile profile with negative slope for the Italian market, and Engström (2001) reports that the smile profile is characterised by a positive slope for the Swedish market. Peña, Rubio, and Serna (1999) show that the smile profile is symmetric in the Spanish market.

There are several reasons cited in the literature for the observable volatility smile relationship. MacBeth and Merville (1979) and Rubinstein (1985) suggest that it may be due to the fact that one or more of the assumptions of the Black-Scholes model are violated in the pricing of actual exchange traded options.

Rubinstein (1994) and Derman and Kani (1994) suggest that the volatility smile can occur if the volatility of an underlying stock or market index is expected to be a function of both time and the level of future stock price, although empirical tests by Dumas, Fleming, and Whaley (1998) cast doubt on this explanation. Hull and White (1987), and Heston (1993) suggest that the volatility smile can be explained by the expectation of stochastic volatility, especially if there is a negative correlation between volatility and the level of future stock prices. Through simulation, Dennis (1996) shows that the volatility smile can occur under standard Black-Scholes assumptions in the presence of transaction costs. The smile asymmetry link to time to maturity and the smile U-shaped pattern are also evidenced for other mature markets (Tompkins, 1999; Peña et al., 1999; Beber, 2001; Engström, 2001).

In contrast, limited research on this subject is available for India owing to the nascent nature of its derivatives market. The previous work provides evidence for the existence of a volatility smile in the Indian options market. Varma (2002) observes mispricing in the Indian Index options market and estimates the volatility smile for call and put options, finding that it is different across option types. Misra, Kannan, and Misra (2006) find that deeply in- and out-of-the-money options have higher implied volatility than at-the-money options and that volatility is higher for far month option contracts than for near month option contracts. The relationship between IV and moneyness seems to have positive asymmetry in the case of NSE Nifty options. These studies fail to provide any strong evidence of volatility smile and determinants of IV because of small sample size, short study periods and immaturity of the market structure in the initial phase. In this paper, we attempt to fill this research gap for the Indian option market literature. This study specifically examines the following propositions:

- (a) Is there an asymmetric volatility smile relationship for the Indian Index option market?
- (b) Does the volatility function vary across different option types, i.e., call and put options?
- (c) What are the key determinants of any observable smile asymmetry?

The study finds that the relationship between IV and moneyness exhibits a positive slope in the Indian context. The smile asymmetry is stronger for put than for call options, and results of the volatility function are consistent with alternative measures of moneyness. The study also observes that the time to expiration and historical volatility are the key determinants of the observable asymmetric profile.

This paper is organised into six sections, including the present one. In the next section, we provide a review of the literature covering IV functions and their determinants. Next, we describe the data and their sources and followed by the tests of the volatility smile relationship and their results. The potential determinants of the volatility smile and their results are provided in the latter section. In the last section, a summary and conclusions are given.

LITERATURE REVIEW

In this section, we provide a brief review of the literature dealing with options volatility smile and determinants of the IV function.

Hull and White (1987) explain an option-pricing problem of a European call on assets having stochastic volatility. Determine the prices of put options non dividend paying stocks in a series form, holding the stochastic volatility independent of the stock price. The study finds the volatility to be correlated with the stock price. The frequent overpricing of options by the Black-Scholes model as well as the degree of overpricing increases with the time to maturity.

Rubinstein (1994) develops a new method for inferring risk-neutral probabilities (or state-contingent prices) from the simultaneously observed prices of S&P 500 index options following the crash of 1987. These probabilities are then used to infer a unique, fully specified recombining binomial tree that is consistent with these probabilities, where a simple backwards recursive procedure solves for the entire tree. Finally, the study observes that "crash-ophobia" causes a slightly bimodal implied distribution to be quite common after the crash. The smile pattern in the implied volatilities prior to the crash changed into a "sneer" in the post-crash period.

Bakshi, Cao, and Chen (1997) examine the pricing and hedging performance of different option pricing models compared to the Black-Scholes model for S&P 500 call options from June 1, 1988 through May 31, 1991. Alternative models are examined from three perspectives: (1) internal consistency of implied parameters/volatility with relevant time series data, (2) out-of-sample pricing, and (3) hedging. The other models allow for stochastic

volatility, stochastic interest rates and jumps in different combinations from the smile pattern evidenced by using the Black-Scholes model. The Black-Scholes IV exhibits a clear U-shaped pattern across moneyness, where the most distinguished smile evident for options happens near expiration.

Dumas et al. (1998) develop a Deterministic Volatility Function (DVF) model for option pricing. This model fits different specifications of the volatility using S&P 500 index options from June 1988 through December 1993. A relatively parsimonious volatility function works well for describing the observed volatility structure, whereas the Black-Scholes constant volatility model appears to be better for determining hedge ratios. However, the results indicate that the volatility functions implied by the option prices are not stable over time.

Peña et al. (1999) analyse the determinants of the smile pattern in IV on the Spanish IBEX-35 index from January 1994 to April 1996. The study performs a simple regression framework together with more sophisticated techniques of both Linear and Nonlinear Granger causality to understand the behaviour of the IV function. The results suggest strong seasonal behaviour in the curvature of the volatility smile, as well as a bidirectional Granger causality between transaction costs (proxied by the bid-ask spread).

Hafner and Wallmeier (2000) examine the pattern of DAX implied volatilities across exercise prices and their determinants using daily call and put prices from 1995 to 1999. A spline regression model with two segments was formulated and the weighted least square method applied. The results show a very accurate fit to the data and demonstrate cross-sectional variation of implied volatilities.

Beber (2001) analyses the potential determinants of the volatility smile using call and put options on the Mib30 Italian stock index from November 1995 to March 1998. The quadratic model of implied volatility in moneyness exhibits the typical smirk. The investigated potential determinants using the linear Granger causality test show a linear causal relation with the time to expiration, the number of transacted option contracts and historical volatility on the asymmetry of the smile profile.

Varma (2002) examines the mispricing of volatility using closing Nifty futures and options prices from June 2001 to February 2002. The study fit a volatility smile and used the Breeden-Litzenberger formula to compute the implied probability distribution for the terminal stock index price from the fitted smile. The implied probability distribution is then compared with the theoretical models to determine whether the observed smile is a reasonable one. The study finds that the market appears to be underestimating the probability of market

movements in either direction and indicates severe underpricing volatility. The study also observes overpricing of deep in-the-money calls and some inconclusive evidence of violation of put-call parity.

Misra et al. (2006) investigate the volatility surfaces and determinants of IV for NSE Nifty options from 1 January 2004 to 31 December 2004. The study reveals that deeply in-the-money and deeply out-of-the-money options' volatility is higher than the at-the-money options; the IV of out-of-the-money call options is more than in-the-money calls; IV is higher for far month contracts than for near month contracts; deeply in-the-money and out-of-the-money options with shorter maturity have higher volatility than those with longer maturity; put options' volatility is higher than call options'; and IV of high liquid options is greater than that of low liquid options. The results provide evidence the existence of the volatility smile in India as experienced by the US market before the stock market crash of 1987.

THE DATA

We employ the S&P CNX Nifty options contract, which is composed of the S&P CNX Nifty Index traded on the NSE derivatives segment. S&P CNX Nifty is a 50 Stock Index (underlying asset) of the NSE comprising the largest and most liquid companies in India, with 60% of the total market capitalisation of the Indian stock market. The Nifty option is a cash settled European option containing a minimum strike price of 9 to a maximum strike price of 13 (4-1-4, 5-1-5 and 6-1-6 viz. ITM-ATM-OTM). Option contracts have a maximum trading cycle of three months – the near month (one), next month (two) and far month (three). The contract expires on the last Thursday of a month or the previous trading day if Thursday is a trading holiday. Trading occurs from 10:00 AM to 3:30 PM. The quoted options premium value is computed based on the Black-Scholes (1973) model; in fact, this is followed by most practitioners (Beber, 2001).

The data comprises Nifty options daily call and put contracts closing prices and trading volumes for near month maturity for the calendar years 2004 and 2005 (the next month and far month option data are not relevant since trading volume are less traded). Weekly implicit yields on 91-day T-bill rates are obtained from the Reserve Bank of India (RBI) website, which is used as a risk-free rate of return for deriving IV. The underlying asset, S&P CNX Nifty Index value, is downloaded from the Indices segment of the NSE website. The daily total trading volumes on S&P CNX Nifty Index traded companies during our study period are obtained from the Equity segment of the NSE. We use daily files involving a wide range of strike prices for both call and put options, which we

collect from the NSE website. We consider only liquid prices of near month options, for which we carried out the following filtering criteria.

First, we deleted the options closing prices with zero transactions. Second, we eliminated the last five trading days to expiration (as in the case of Beber, 2001) and considered options data only for near month. The objective is to remove liquidity bias owing to excess trading volume close to the expiration date. We eliminate very deep in-the-money and very deep out-of-the-money option contracts with strike price differences from spot value of 100 or more. This filtering procedure results in data for 418 and 395 trading days for call and put option contracts, respectively. Thus, we give a more involved and active part in our work to in-the-money, at-the-money and out-of-the-money call and put options for computing IV.

TESTS OF IV FUNCTION

The volatility smile is a variation of IV with respect to options moneyness. The estimation of volatility smile basically involves evaluating the relationship between the IV and the level of moneyness for any given option.

The Variables

IV is inferred in the market from option prices using a suitable option pricing model. It is a measure of the amount and speed of price change in either direction. This indicates the expensiveness of option premium for the traders.

We compute IV (σ_{it}) by solving the Black-Scholes (1973) formula for each observed European call (C_{it}) and put (P_{it}) option closing price as shown in Equations (1) and (2):

$$C_{it} = S_0 (N d_1) - Ke^{-rT} N(d_2) \quad (1)$$

$$P_{it} = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \quad (2)$$

where

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma_{it}^2 / 2)T}{\sigma_{it} \sqrt{T}}$$

$$d_2 = d_1 - \sigma_{it} \sqrt{T}$$

The function $N(x)$ is the cumulative probability distribution function. The variable S_0 is the stock index price at time zero, K is the strike price, r is the continuously compounded risk-free rate of return (91 day T-bill yield rate), and T is the time to expiration of the option.

We define moneyness as the ratio between the exercise price and the average of the option prices relative to each average IV. Moneyness can be computed in three ways: First is the ratio of options strike price to the underlying Index price (Jackwerth & Rubinstein, 1996). A modified version of this measure can be estimated as the absolute value of the difference between index value and strike price divided by Index value (called M1 hereafter), i.e., $|S-K/S|$ (Misra et al., 2006). This method, though simple, does not consider the volatility of the underlying asset and the option's time to maturity. Incorporating these elements, one can estimate moneyness using an alternative measure (hereafter referred to as M2) by taking the natural logarithm of the ratio of the strike price to the underlying Index value and then dividing it by the product of at-the-money IV and the square root of the time to maturity as shown in Equation (3) (Natenberg, 1994; Dumas et al., 1998; Tompkins, 1999; Beber, 2001).

$$\frac{\ln\left(\frac{K_{it}}{S_t}\right)}{\sigma_{atm,t}\sqrt{T-t}} \quad (3)$$

To compute at-the-money IV, we use the average of IV of a call and a put option with a strike price K^* as close as possible to the index level as shown below:

$$K^* = \arg_k \min [S_t/K - 1]$$

The third approach to compute moneyness is to use directly the options' *Delta*. This measure (hereafter referred to as M3) considers both time to maturity and volatility and is coherent with the Black-Scholes model. To avoid dependencies between the measures of moneyness and IV of options with different strike prices, at-the-money IV is inserted in the *Delta*'s formula for each option as the volatility input.

It is convenient to normalise options' *Delta* between zero and one in order to represent call and put options in the same way; this can be achieved by taking the absolute value of a put option's *Delta* and the complement to one of the call options' *Delta*, so as to obtain normalised values that increase with the strike price (Beber, 2001).

Beber (2001) estimates only the second and third measures of moneyness using data on future price for the Italian MIB 30 stock market index. He uses futures market data instead spot market data owing to the fact that the MIB-30 index is non-synchronous with the futures and options markets in Italy with regard to their closing times. Such a problem does not exist in the Indian context, and therefore we employ spot market data for estimating moneyness. In our study, we use all three measures of moneyness described above, namely M1, M2 and M3.

Model Specification and Estimation

We next estimate the IV function to evaluate the volatility smile relationship in the Indian environment. Before examining the volatility smile relationship, we attempt to determine a perfect model that fits options' IV against their moneyness. We avoid classification of moneyness as in Beber (2001) because it is not useful for further analysis in our work. We are simply interested in fitting the IV across moneyness on a period-to-period basis.

We estimate the IV function using linear and quadratic models given in Equations (4) and (5) as suggested by previous literature (Shimko, 1993; Dumas et al., 1998; Beber, 2001):

$$\text{Model 1: } Y = \beta_0 + \beta_1 X + \varepsilon \quad (4)$$

$$\text{Model 2: } Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \varepsilon \quad (5)$$

where Y represents the IV and X represents moneyness of the options. The simplicity in the case of two models is maintained in an endeavour to avoid overparameterisation and obtain better estimates.

We give equal weight to each observation, regardless of moneyness, as a strategy to assign less weight to the deep out-of-the-money options owing to higher volatility which has not proven to be satisfactory (Jackwerth & Rubinstein, 1996). However, these models can hardly be estimated on the given dataset because the general level of volatility is time-varying.

We adopt an estimation procedure to take care of this problem. Specifically, we fit the models separately on every trading day with sufficient observations; we implicitly assume that IV is stationary during the day. Then, in order to obtain representative model parameters for the whole period, the average of the daily estimates is computed. We do not need an at-the-money IV estimate, and information about the general level of volatility is retained.

Hence, we rearrange the above-said model to consider this approach:

$$\text{Model 1: } Y_{t,\tau} = \beta_0 + \beta_1 X_{t\tau} + \varepsilon_{t\tau} \quad (6)$$

$$\text{Model 2: } Y_{t,\tau} = \beta_0 + \beta_1 X_{t\tau} + \beta_2 X_{t\tau}^2 + \varepsilon_{t\tau} \quad (7)$$

where Y represents IV and X represents the measure of moneyness, while t denotes trading days and τ options' time to maturity. A single model is estimated for each trading day, considering options with the same time to maturity; typically, two forms of the model are estimated for each trading day (linear and quadratic).

The results obtained by Ordinary Least Square (OLS) fitting for the linear model are shown in Table 1. The reported parameter estimates are averages for the whole period, as is the adjusted R^2 . The t -statistics have been computed using the average value of the parameter and average standard error, instead of averaging the t -values of each parameter. The mean of the intercept (β_0) of the model given in Equation (6) represents a general level of volatility that localises the IV function. The mean slope, i.e., β_1 characterises the profile that is responsible for the asymmetry in the risk neutral probability function. Using the first two measures of moneyness, M1 and M2, we find β_1 to be strongly positive for call as well as put options. Employing the third measure of moneyness (M3), which is the delta based measure, we find β_1 to be statistically insignificant for call options, while it is strongly positive (as in the case of M1 and M2 measures) for put options. In general, the option volatility function exhibits positive asymmetry, which is in contrast with the findings of most mature markets (Rubinstein, 1994; Jackwerth & Rubinstein, 1996; Dumas et al., 1998; Beber, 2001). One possible explanation for the observed positive asymmetry could be the possible biases in building expectations of stochastic volatility, especially if there is a positive correlation between volatility and the level of future stock prices.

In the case of M3 (delta based measure), another possible explanation could be the violation of some of the assumptions of the Black-Scholes model in the pricing of actual exchange-traded options, including transaction costs. Further, the positive asymmetry profile is stronger for put compared to call options. This may be owing to asymmetry in trading costs, as the former is less liquid than the latter in the Indian environment. Our results are consistent across alternative measures of moneyness, thus confirming the robustness of the options smile relationship.

Table 1
The Relationship Between IV and Moneyness: Fitting Linear Model

Moneyness measure (M1)					
Options	β_0	β_{0t}	β_1	β_{1t}	R^2
Call	0.156	13.614*	0.867	2.883*	0.361
Put	0.119	1.966*	11.383	2.989*	0.667

Moneyness measure (M2)					
Options	β_0	β_{0t}	β_1	β_{1t}	R^2
Call	0.201	7.988*	0.154	6.071*	0.211
Put	0.194	1.804	11.515	107.350*	0.647

Moneyness – Delta measure (M3)					
Options	β_0	β_{0t}	β_1	β_{1t}	R^2
Call	0.209	8.446*	-0.003	-0.132	0.318
Put	0.384	4.110*	1.417	15.176*	0.966

Note: The table presents ordinary regression results for different measures of moneyness, M1, M2 and M3, both for call and put options.

* denotes significance at the 5% level for all tables

We find that a quadratic model does not provide a better explanation of the IV function, thus ruling out any curvature in the IV moneyness relationship. Therefore, we do not discuss the results related to the quadratic model in this paper.

POTENTIAL DETERMINANTS OF VOLATILITY SMILE

In this section, we define the potential determinants of volatility smile and test their relationships with the observed asymmetry profile.

The Variables

We categorise the variables into three groups: First, we explain the specific features of the option market in order to obtain a reasonable estimate of general liquidity and potential links between positive asymmetry and IV term structure. Thus, our aim is to explain options' mispricing with the existence of market frictions. Second, we represent the underlying asset dynamic in order to detect likely dependencies between the IV function and relevant characteristics of the underlying asset, where options' mispricing is explained by an investor

assessment of the underlying stochastic process, which is different from the Black-Scholes hypothesis. Finally, we try to capture investors' behaviour in order to reflect market practices that could have an impact on the IV function as a consequence of relative trading activity in calls *versus* puts of all strike prices. Thus, as argued by option market practitioners, heavy demand for out-of-the-money put options drives up prices.

In the first category of options market specific variables, we employ the option's residual time to expiration calculated as the ratio between the number of working days to the expiration date and the conventional working days in a year:

$$TEXP_t = \frac{\text{expiration date} - \text{trading date}}{252} \quad (8)$$

Clearly, we try to account for the potential effect of the time horizon on the implied risk neutral density function.

We use daily options trading volume, summed for all strike prices, as a measure of option market liquidity:

$$NOPT = \sum_{i=1}^K n_{it} \quad (9)$$

where n_i is the number of contracts traded on i -option on day t .

The second category of explanatory variables representing an underlying asset's dynamics are the market momentum, volume of market transactions, historical volatility and volatility of volatility.

The momentum is calculated as the natural logarithm of the ratio between the index value (S&P CNX Nifty) and its 21-day simple moving average:

$$MOM_t = \ln \frac{S_t}{\frac{1}{21} \sum_{i=t-20}^t S_i} \quad (10)$$

where S_t is the index value at time t . This ratio will be high (low) in a bullish (bearish) market. Even if this measure is somewhat arbitrary, the aim is to gauge if the underlying trend has any effects on the steepness of the smile profile, given

that it is well known that a *leverage* effect is present for the general level of volatility (e.g., Schwert, 1989).

The volume of market transactions is the second potential explanatory variable, expressed as the total daily trading volume for all stocks that form part of the S&P CNX Nifty Index (NIVOL). This liquidity proxy is used as a measure of the general market activity in the underlying asset. Third, we consider historical volatility, which is calculated during the previous 14 trading days as the annualised standard deviation of logarithmic returns on S&P CNX Nifty settlement prices:

$$HVOL_t = [\sigma (r_t, \dots, r_{t-13})] \sqrt{252} \quad (11)$$

where $\sigma (x, \dots, y)$ is the standard deviation of values between x and y and r_t is the daily logarithmic return on S&P CNX Nifty settlement prices. The potential effects of different levels of volatility on the smile profile would suggest that an obvious pricing improvement is to relax the assumption of constant volatility.

The last variable employed in the second category is the volatility of volatility, computed during the previous 14 trading days like the standard deviation of the historical volatility shown before:

$$VVOL_t = [\sigma (HVOL_t, \dots, HVOL_{t-13})] \quad (12)$$

where $\sigma (x, \dots, y)$ is the standard deviation of values between x and y . This variable is a good proxy for the *vega* risk in hedging activity, which may affect the pricing of certain types of options.

Investors' behaviour is the third category of potential determinants, comprising one variable: The number of contracts written on out-of-the-money put options as a percentage of total reported out-of-the-money call and put transactions, in order to obtain a sort of measure of the portfolio insurance activity of fund managers (Beber, 2001):

$$VPUT_t = \frac{\sum_{i=1}^k nput_t}{\sum_{i=1}^m n_i} \quad (13)$$

where n_i is the usual number of contracts traded on i -option (call plus put) and $nput_t$ is the number of contracts traded on i -put option, where k of the total m out of the money options are puts.

We check the stationarity for all these variables using the Augmented Dickey Fuller (ADF) test (unit root test). The ADF test suggests that these variables are stationary at the 5% level except NOPT, which is stationary at the first difference and hence is integrated to the order one.

We also estimate the correlations between the explanatory variables, which are presented in Table 2. We find high correlations between underlying asset dynamic variables like historical volatility, momentum, volatility of volatility and Index volumes. It appears that historical volatility can capture the impact of most of the underlying asset variables in our endeavour to explain smile asymmetry. However, we show our results for all variables since they are used independently in causality tests, which are discussed in the next sub-section.

The Causality Tests

In the previous section we discussed the best fit model for the IV function. In this section we try to identify the explanatory variable that potentially determines the IV smile profile. We perform linear and non-linear Granger causality tests between the estimated slope parameter (β_1) of Model 1 and the potential determinants described above in this section. In some sense, the contract-specific variables would help to detect cross sectional pricing biases while the other variables serve to detect the asymmetry profile over time, which means the Black-Scholes pricing error is related to the dynamically changing market conditions.

The causality test captures the bi-directional relationship between two variables, say, Y_t and X_t (Y_t causes X_t and X_t causes Y_t), with distributed lags of the VAR model. To find this cause effect relationship, we perform Granger's linear causality test (1969). The Granger causality test involves testing of the null hypotheses ' X_t does not cause Y_t ' and ' Y_t does not cause X_t ' by simply running the

Table 2
Correlation Matrix for Potential Determinants of Smile Asymmetry.

	TEXP	NOPT	NOPTLAG1	MOM	HVOL	VVOL	NIVOL	VPUT
TEXP	1							
NOPT	-0.084	1						
NOPTLAG1	0.059	-0.088	1					
MOM	-0.107	0.000	0.091	1				
HVOL	-0.035	-0.181	0.050	0.442	1			
VVOL	-0.068	-0.131	-0.044	0.298	0.716	1		
NIVOL	-0.035	-0.316	-0.178	0.185	0.350	0.165	1	
VPUT	0.008	-0.126	-0.035	-0.047	0.004	0.028	0.029	1

following two regression estimations of VAR model by employing Schwarz information criteria for lag length 4 in our case.

$$Y_t = a_1 + \sum_{i=1}^n \beta_i \chi_{t-i} + \sum_{i=1}^n \gamma_i Y_{t-i} + e_{1t} \quad (14)$$

$$X_t = a_2 + \sum_{i=1}^n \theta_i \chi_{t-i} + \sum_{i=1}^n \delta_i Y_{t-i} + e_{2t} \quad (15)$$

where it is assumed that the lagged values of Y_t and X_t are uncorrelated white noise error terms.

Then, the reported results are interpreted using F-statistics, which is a valid test for joint hypotheses. While determining the lag length, we carefully follow Schwarz information criteria instead of Akaike information criteria, since the Schwarz is more parsimonious and stable to control over parameterisation of the variables (Beber, 2001).

The application of the linear causality test may provide a weak result and fail to capture the nonlinear dependencies of the employed variables. Therefore, we apply the vector error correction Granger causality test (Granger's non-linear causality test) between β_1 and the seven potential determining variables. In order to capture such a nonlinear relationship, similar tests are performed by Baek and Brock (1992), Hiemstra and Jones (1994), Abhyankar (1998), Peña et al. (1999), and Beber (2001).

The results of the relationship between the asymmetry profile and its potential determining variables for alternative measures of moneyness M1, M2, and M3 are provided in Tables 3–5. Panel A and B of each table contain results of linear and non-linear Granger causality tests for call and put options, respectively. Based on the linear causality test, we find that TEXP and HVOL cause smile asymmetry in the case of call options. While the former variable captures the impact of time on IV, the latter may be responsible for the cross sectional differences in volatility for any given day. We choose to ignore the causal behaviour of the MOM factor, owing to the fact that it is significantly correlated with HVOL and hence can be captured by the latter. We do not interpret results based on the M3 measure as showing that the volatility smile is symmetric for call option data. The Granger's nonlinear causality test does not reveal any additional variables that disproportionately explain the asymmetry profile. The results for put options based on linear as well as nonlinear causality tests are similar to those for call options, given the overlapping nature (see Table 2) of HVOL and other underlying asset variables.

Table 3

Potential Determinants of Smile Asymmetry based on First Measure of Moneyness (M1): Granger Linear and Nonlinear Causality Tests.

Panels A and B provide results of the linear and nonlinear Granger causality tests for call and put options, respectively.

Panel A. Results for the linear Granger causality test

Call option			
Causal direction	L	F-statistic	P-value
TEXP \rightarrow β_1	4	8.116*	0.000
NOPT ¹ \rightarrow β_1	4	0.666	0.616
MOM \rightarrow β_1	4	0.349	0.844
NIVOL \rightarrow β_1	4	0.809	0.520
HVOL \rightarrow β_1	4	0.969	0.424
VVOL \rightarrow β_1	4	0.636	0.637
VPUT \rightarrow β_1	4	0.723	0.576
Causal direction	L	F-statistic	P-value
$\beta_1 \rightarrow$ TEXP	4	1.894	0.111
$\beta_1 \rightarrow$ NOPT ¹	4	0.975	0.421
$\beta_1 \rightarrow$ MOM	4	1.255	0.287
$\beta_1 \rightarrow$ NIVOL	4	1.445	0.218
$\beta_1 \rightarrow$ HVOL	4	1.232	0.297
$\beta_1 \rightarrow$ VVOL	4	0.078	0.988
$\beta_1 \rightarrow$ VPUT	4	0.035	0.997

Put option			
Causal direction	L	F-statistic	P-value
TEXP \rightarrow β_1	4	3.990*	0.003
NOPT ¹ \rightarrow β_1	4	0.743	0.563
MOM \rightarrow β_1	4	1.462	0.213
NIVOL \rightarrow β_1	4	0.053	0.995
HVOL \rightarrow β_1	4	1.452	0.216
VVOL \rightarrow β_1	4	0.266	0.899
VPUT \rightarrow β_1	4	0.529	0.714
Causal direction	L	F-statistic	P-value
$\beta_1 \rightarrow$ TEXP	4	11.960*	0.000
$\beta_1 \rightarrow$ NOPT ¹	4	0.543	0.704
$\beta_1 \rightarrow$ MOM	4	0.615	0.652
$\beta_1 \rightarrow$ NIVOL	4	0.119	0.078
$\beta_1 \rightarrow$ HVOL	4	1.654	0.160
$\beta_1 \rightarrow$ VVOL	4	0.552	0.697
$\beta_1 \rightarrow$ VPUT	4	0.742	0.564

Note: L denotes the optimal lag with Schwarz information criteria for all the cases of linear Granger causality test

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Table 3 (continued)

Panel B. Results for the VEC (nonlinear) Granger causality test

Call option			
Causal direction	df	Chi-sq.	P-value
TEXP \rightarrow β_1	4	29.938*	0.000
NOPT ¹ \rightarrow β_1	4	5.637	0.228
MOM \rightarrow β_1	4	13.644*	0.008
NIVOL \rightarrow β_1	4	3.830	0.429
HVOL \rightarrow β_1	4	2.479	0.648
VVOL \rightarrow β_1	4	3.278	0.512
VPUT \rightarrow β_1	4	2.118	0.714
Causal direction	df	Chi-sq.	P-value
$\beta_1 \rightarrow$ TEXP	4	6.512	0.164
$\beta_1 \rightarrow$ NOPT ¹	4	0.962	0.916
$\beta_1 \rightarrow$ MOM	4	22.724*	0.000
$\beta_1 \rightarrow$ NIVOL	4	5.012	0.286
$\beta_1 \rightarrow$ HVOL	4	3.148	0.533
$\beta_1 \rightarrow$ VVOL	4	2.673	0.614
$\beta_1 \rightarrow$ VPUT	4	1.578	0.813
Put option			
Causal direction	df	Chi-sq.	P-value
TEXP \rightarrow β_1	4	38.000	0.000
NOPT ¹ \rightarrow β_1	4	5.547	0.236
MOM \rightarrow β_1	4	2.259	0.688
NIVOL \rightarrow β_1	4	2.970	0.563
HVOL \rightarrow β_1	4	1.995	0.737
VVOL \rightarrow β_1	4	1.488	0.829
VPUT \rightarrow β_1	4	7.506	0.111
Causal direction	df	Chi-sq.	P-value
$\beta_1 \rightarrow$ TEXP	4	52.748*	0.000
$\beta_1 \rightarrow$ NOPT ¹	4	4.690	0.320
$\beta_1 \rightarrow$ MOM	4	2.621	0.623
$\beta_1 \rightarrow$ NIVOL	4	2.147	0.704
$\beta_1 \rightarrow$ HVOL	4	5.144	0.273
$\beta_1 \rightarrow$ VVOL	4	2.014	0.733
$\beta_1 \rightarrow$ VPUT	4	6.651	0.155

Note: df denotes (the optimal lag with Schwarz information criteria) degree of freedom for a Chi-squared distribution for all the cases of VEC (nonlinear) Granger causality test

Table 4

Potential Determinants of Smile Asymmetry based on Second Measure of Moneyiness (M2): Granger Linear and Nonlinear Causality Tests.

Panels A and B provide results of the linear and nonlinear Granger causality tests for call and put options, respectively.

Panel A. Results for the linear Granger causality test

Call option			
Causal direction	L	F-statistic	P-value
TEXP \rightarrow β_1	4	3.793*	0.004
NOPT ¹ \rightarrow β_1	4	0.898	0.464
MOM \rightarrow β_1	4	1.309	0.266
NIVOL \rightarrow β_1	4	0.467	0.760
HVOL \rightarrow β_1	4	2.385*	0.050
VVOL \rightarrow β_1	4	0.987	0.414
VPUT \rightarrow β_1	4	1.056	0.378
Causal direction	L	F-statistic	P-value
$\beta_1 \rightarrow$ TEXP	4	1.124	0.345
$\beta_1 \rightarrow$ NOPT ¹	4	0.967	0.425
$\beta_1 \rightarrow$ MOM	4	4.220*	0.002
$\beta_1 \rightarrow$ NIVOL	4	0.837	0.503
$\beta_1 \rightarrow$ HVOL	4	2.483*	0.043
$\beta_1 \rightarrow$ VVOL	4	3.810*	0.005
$\beta_1 \rightarrow$ VPUT	4	1.697	0.149

Put option			
Causal direction	L	F-statistic	P-value
TEXP \rightarrow β_1	4	1.256	0.287
NOPT ¹ \rightarrow β_1	4	0.619	0.649
MOM \rightarrow β_1	4	1.967	0.099
NIVOL \rightarrow β_1	4	2.551*	0.039
HVOL \rightarrow β_1	4	10.259*	0.000
VVOL \rightarrow β_1	4	1.810	0.126
VPUT \rightarrow β_1	4	0.980	0.418
Causal direction	L	F-statistic	P-value
$\beta_1 \rightarrow$ TEXP	4	1.340	0.254
$\beta_1 \rightarrow$ NOPT ¹	4	0.901	0.463
$\beta_1 \rightarrow$ MOM	4	1.101	0.355
$\beta_1 \rightarrow$ NIVOL	4	0.231	0.920
$\beta_1 \rightarrow$ HVOL	4	4.497*	0.001
$\beta_1 \rightarrow$ VVOL	4	4.367*	0.001
$\beta_1 \rightarrow$ VPUT	4	0.344	0.48

Note: L denotes the optimal lag with Schwarz information criteria for all the cases of linear Granger causality test

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Table 4 (continued)

Panel B. Results for the VEC (nonlinear) Granger causality test

Call option			
Causal direction	df	Chi-sq.	P-value
TEXP \rightarrow β_1	4	13.174*	0.004
NOPT ¹ \rightarrow β_1	4	3.563	0.468
MOM \rightarrow β_1	4	5.239	0.264
NIVOL \rightarrow β_1	4	1.869	0.760
HVOL \rightarrow β_1	4	9.470	0.049
VVOL \rightarrow β_1	4	3.947	0.413
VPUT \rightarrow β_1	4	4.226	0.376
Causal direction	df	Chi-sq.	P-value
$\beta_1 \rightarrow$ TEXP	4	4.496	0.434
$\beta_1 \rightarrow$ NOPT ¹	4	3.869	0.424
$\beta_1 \rightarrow$ MOM	4	16.880*	0.002
$\beta_1 \rightarrow$ NIVOL	4	3.347	0.501
$\beta_1 \rightarrow$ HVOL	4	9.932*	0.042
$\beta_1 \rightarrow$ VVOL	4	15.242*	0.004
$\beta_1 \rightarrow$ VPUT	4	6.792	0.147
Put option			
Causal direction	df	Chi-sq.	P-value
TEXP \rightarrow β_1	4	5.024	0.284
NOPT ¹ \rightarrow β_1	4	2.477	0.649
MOM \rightarrow β_1	4	7.689	0.096
NIVOL \rightarrow β_1	4	10.202*	0.037
HVOL \rightarrow β_1	4	41.037*	0.000
VVOL \rightarrow β_1	4	7.243	0.123
VPUT \rightarrow β_1	4	3.921	0.417
Causal direction	df	Chi-sq.	P-value
$\beta_1 \rightarrow$ TEXP	4	5.361	0.252
$\beta_1 \rightarrow$ NOPT ¹	4	3.605	0.426
$\beta_1 \rightarrow$ MOM	4	4.407	0.54
$\beta_1 \rightarrow$ NIVOL	4	0.926	0.921
$\beta_1 \rightarrow$ HVOL	4	17.989*	0.001
$\beta_1 \rightarrow$ VVOL	4	17.468*	0.002
$\beta_1 \rightarrow$ VPUT	4	1.376	0.848

Note: df denotes (the optimal lag with Schwarz information criteria) degree of freedom for a Chi-squared distribution for all the cases of VEC (nonlinear) Granger causality test

Table 5
*Potential Determinants of Smile Asymmetry based on Third Measure of Moneyness (M3):
 Granger Linear and Nonlinear Causality Tests.*

Panels A and B provides results of the linear and nonlinear Granger causality tests for call and put options, respectively.

Panel A. Results for the linear Granger causality test

Call option			
Causal direction	L	F-statistic	P-value
TEXP \rightarrow β_1	4	–	–
NOPT ¹ \rightarrow β_1	4	–	–
MOM \rightarrow β_1	4	–	–
NIVOL \rightarrow β_1	4	–	–
HVOL \rightarrow β_1	4	–	–
VVOL \rightarrow β_1	4	–	–
VPUT \rightarrow β_1	4	–	–

Causal direction	L	F-statistic	P-value
$\beta_1 \rightarrow$ TEXP	4	–	–
$\beta_1 \rightarrow$ NOPT ¹	4	–	–
$\beta_1 \rightarrow$ MOM	4	–	–
$\beta_1 \rightarrow$ NIVOL	4	–	–
$\beta_1 \rightarrow$ HVOL	4	–	–
$\beta_1 \rightarrow$ VVOL	4	–	–
$\beta_1 \rightarrow$ VPUT	4	–	–

Put option			
Causal direction	L	F-statistic	P-value
TEXP \rightarrow β_1	4	0.342	0.849
NOPT ¹ \rightarrow β_1	4	1.823	0.123
MOM \rightarrow β_1	4	7.981*	0.000
NIVOL \rightarrow β_1	4	1.281	0.267
HVOL \rightarrow β_1	4	7.279*	0.004
VVOL \rightarrow β_1	4	7.545	0.000
VPUT \rightarrow β_1	4	0.851	0.493

Causal direction	L	F-statistic	P-value
$\beta_1 \rightarrow$ TEXP	4	3.352*	0.010
$\beta_1 \rightarrow$ NOPT ¹	4	1.461	0.213
$\beta_1 \rightarrow$ MOM	4	0.020	0.090
$\beta_1 \rightarrow$ NIVOL	4	0.746	0.561
$\beta_1 \rightarrow$ HVOL	4	3.921*	0.001
$\beta_1 \rightarrow$ VVOL	4	10.515*	0.000
$\beta_1 \rightarrow$ VPUT	4	1.861	0.116

Note: L denotes the optimal lag with Schwarz information criteria for all the cases of linear Granger causality test

(continued on next page)

Table 5 (continued)

Panel B. Results for the VEC (nonlinear) Granger causality test

Call option			
Causal direction	df	Chi-sq.	P-value
TEXP $\rightarrow \beta_1$	4	–	–
NOPT ¹ $\rightarrow \beta_1$	4	–	–
MOM $\rightarrow \beta_1$	4	–	–
NIVOL $\rightarrow \beta_1$	4	–	–
HVOL $\rightarrow \beta_1$	4	–	–
VVOL $\rightarrow \beta_1$	4	–	–
VPUT $\rightarrow \beta_1$	4	–	–

Causal direction	df	Chi-sq.	P-value
$\beta_1 \rightarrow$ TEXP	4	–	–
$\beta_1 \rightarrow$ NOPT ¹	4	–	–
$\beta_1 \rightarrow$ MOM	4	–	–
$\beta_1 \rightarrow$ NIVOL	4	–	–
$\beta_1 \rightarrow$ HVOL	4	–	–
$\beta_1 \rightarrow$ VVOL	4	–	–
$\beta_1 \rightarrow$ VPUT	4	–	–

Put option			
Causal direction	df	Chi-sq.	P-value
TEXP $\rightarrow \beta_1$	4	2.935	0.568
NOPT ¹ $\rightarrow \beta_1$	4	8.705	0.069
MOM $\rightarrow \beta_1$	4	31.924*	0.000
NIVOL $\rightarrow \beta_1$	4	7.074	0.132
HVOL $\rightarrow \beta_1$	4	0.862	0.425
VVOL $\rightarrow \beta_1$	4	6.000	0.199
VPUT $\rightarrow \beta_1$	4	5.119	0.275

Causal direction	df	Chi-sq.	P-value
$\beta_1 \rightarrow$ TEXP	4	15.258*	0.004
$\beta_1 \rightarrow$ NOPT ¹	4	2.886	0.577
$\beta_1 \rightarrow$ MOM	4	8.083	0.088
$\beta_1 \rightarrow$ NIVOL	4	7.469	0.113
$\beta_1 \rightarrow$ HVOL	4	41.967*	0.000
$\beta_1 \rightarrow$ VVOL	4	50.120*	0.000
$\beta_1 \rightarrow$ VPUT	4	7.644	0.105

Note: df denotes (the optimal lag with Schwarz information criteria) degree of freedom for a Chi-squared distribution for all the cases of VEC (nonlinear) Granger causality test

Our results on potential determinants of smile asymmetry are more or less consistent with mature markets. However, option trading volume does not seem to be an important explanatory variable in the Indian context as is the case in Beber (2001). Our results on positive smile asymmetry must be interpreted in light of the fact that the study period (2004–2005) witnessed a strong market upturn. Hence, the market activity was dominated by momentum traders (including speculators), who treat options as an alternative investment to underlying assets and thus ignore the former for their hedging role. Lack of hedging activity on the part of investors leads to relative illiquidity at higher levels of moneyness, causing the option premium to be large and more volatile, as we observe in the Indian data. In contrast, the negative asymmetry observed in the US and other mature markets, especially after the October 1987 crash, may be an outcome of increased hedging activity due to more risk-averse behaviour.

SUMMARY AND CONCLUSION

In this paper, we examine two related propositions for the Indian equity options market:

1. The relationship between IV and option moneyness typically referred to as volatility smile.
2. The potential determinants of the volatility function.

We used daily data for the S&P CNX Nifty Index option as well as the underlying market index for the calendar years 2004–2005. We employed three measures of options moneyness:

- (a) The absolute value of the difference between index value and strike price divided by index value.
- (b) The natural logarithm of the ratio of the strike price on the underlying Index value divided by the product of at-the-money IV and the square root of the time to maturity.
- (c) A delta measure provided by the Black-Scholes (1973) option pricing model.

We find that the IV-moneyness relationship is explained by a linear model. In general, there seems to be a positive asymmetry profile for both call and put options. Our results on smile asymmetry are robust to alternative constructions of the moneyness measure. The positive asymmetry is, however, stronger for put options compared to call options. The positive asymmetry may

be an outcome of the violation of the Black-Scholes (1973) model owing to high transaction cost¹ and restrictions on short selling in the Indian context. It may also be caused by expectations about stochastic volatility, which may be impacted by the positive correlation between IV and the level of futures index value, as we observed.

Our findings are in contrast with those for mature markets, where the empirical evidence largely shows a negative asymmetry profile. This difference may reflect the contrast in underlying investor behaviour, i.e., the Indian investors use options as an alternative asset rather than a hedging mechanism over the study period, given the bull run in market performance; while US and other international investors have become relatively more risk averse and hence use options as a hedging tool after the stock market crisis of October 1987. Additionally, the slope difference may also partially be explained by reasons such as market microstructure issues (trading cost, short selling restrictions, etc.) and differences in investors' expectations about stochastic volatility as stated earlier.

We also attempted to find the potential determinants of the observable asymmetry profile. The choice of descriptors was guided by similar work by Beber (2001) in this area. We find that TEXP and HVOL linearly cause the asymmetry profile for both call and put options, as was the case for mature markets. While the former variable captures the impact of time on IV, the latter may be responsible for the cross sectional differences in IV for any given day.

Although our causality tests are able to detect dependencies with high power, they do not provide any guidance regarding the source of this dependence. We need to provide a theory that takes into account the empirical evidence. Perhaps investment researchers have to travel a distance before all Black-Scholes biases are explained. Hence, investors may continue to use this popular option pricing model, but they should be cognisant of the observable limitations.

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¹ In India, the transaction costs on options vary; that is, they become higher as we move farther away from an at-the-money position, which is marked by a sharp decline in options trading volume.

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