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EFFECTIVENESS OF CLASSIC VERSIONS OF OPTIONS PRICING MODELS IN RECENT WAVES OF FINANCIAL UPHEAVALS

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ABSTRACT

This paper attempts to determine the best alternative model of options pricing with the capacity to control both the level of skewness and kurtosis. It aims to replicate the effectiveness of classic stochastic and deterministic option pricing models and also establish a correlation between the underlying stock returns and their volatility. The paper follows a structural approach for analysing the Hull-White model (with two stochastic versions: non-related and correlated) with respect to the Black-Scholes model, which is a benchmark model. The focus is on fabricating such a model for predicting and protecting the market options price during uncertain financial upheavals. The suggested models have been tested in extreme conditions to determine effectiveness. Furthermore, the paper also examines the hedging effectiveness of hypothecated models.

Keywords: Black-Scholes, European call options, Hull-White, Nifty index, Merton

INTRODUCTION

Based on the framework of geometric Brownian motion and its counterpart Wiener (1938) process, Black and Scholes (1973) and Merton (1973) derived the formula for pricing European call options. The Black-Scholes-Merton (BSM) model assumes that the underlying assets follow a log-normal distribution pattern with constant drift and diffusion. The BSM model also assumes that the parameters *viz.* drift and diffusion (volatility) remain constant throughout the life of the option. However, in reality, the juxtaposition of implied volatility, moneyness and maturity manifests a smile. Further analysis revealed that the non-normality of the return distribution of the underlying assets and volatility clustering (serial autocorrelation of asset return) causes smile (Black, 1975; Cox & Ross, 1976; Johnson & Shanno, 1987; Merton, 1976a, 1976b; Scott, 1987; Stein & Stein, 1991, Wiggins, 1987). The flock concatenated smile to orderly mispricing of options across moneyness and maturity. Later, the cause of the pricing error of Black-Scholes (BS) model was revealed to be rooted mainly in the model itself. The unrealistic theoretical assumptions of the model, namely financial characteristics such as non-normal distribution of the asset's return (Figure 1), leverage effect (Figure 2) and volatility clustering (Figure 3), found in almost all tradable financial asset return series data, were responsible for the pricing bias.

Theoretical and empirical discrepancies of BS triggered the development of more advanced and complex stochastic volatility models, which focus on random dynamics and the interplay of asset returns, including return volatility. These models were based on the assumption that, for complex volatility models, the solution of the Partial Differential Equation (PDE) of BSM would not be a difficult task. However, this was not the case, as the solution of a PDE in a stochastic framework mainly depends on the risk preferences of traders; creation of a riskless portfolio was not possible when only an option and its underlying assets were available for trading. However, in cases when volatility is also available for trading, then perfect hedging would be achieved, and the criteria of a risk free solution for a PDE could thus be met. Therefore, to mitigate the risk of volatility, risk premium was introduced into the theory of options pricing. Scott (1987), Johnson and Shanno (1987), Wiggins (1987), and Hull and White (1987, 1988) generalised the framework of Black-Scholes and set the foundation for the development of stochastic models in options pricing.

The stochastic volatility model, proposed by Scott (1987), is based on the assumption that volatility follows a continuous diffusion process. Johnson and Shanno (1987) assumed a correlation between stock returns and return volatility, whereas Wiggins (1987) assumed a hopscotch finite difference method. To incorporate the random dynamics of asset pricing into option pricing models, Hull-White (1987) focused on the development of a more realistic and appropriate pricing method/model and achieved first success in developing a scientific approach to value 'options'. By modelling the inter-dynamics of asset pricing and its volatility, they had laid the foundation for new era in option pricing. Empirical studies have revealed that the models that incorporate a correlation between asset price and return volatility are more consistent with fatter tails in the asset return distribution and are thus closer to reality. To price options, HW87 employs Taylor's power series approximation on the average value of the stochastic variance of assets. The model of HW87 is based on the assumption that not only the return of the assets, but also their volatility, is stochastic (random) in nature. Hull and White hypothesised that, when volatility is itself volatile, available information is of no use to traders because is insufficient for determining future levels of volatility. This assumption created a dilemma for traders and investors, as it invalidated the conceptual framework of fundamental and technical analysis (indirectly). Hull and White further stated that, in cases when asset price follows a random volatility process, the investors are exposed to a risk in addition to the risk of random evolvement of asset price process. This scenario implies that option price can change even if there is no change in the price of its underlying assets during the life of an option because the volatility process alone is strong enough to change the option price. To provide an appropriate model for option pricing, Hull-White conceptualised another framework. Whereas the first version modelled an asset's price and its volatility as a stochastic process and assumed that there is no correlation between them, in the second framework they assumed a correlation between the two processes and extended the first framework to incorporate the leptokurtic behaviour of asset return and smile together. However, contrary to their earlier counterparts, they did not allow negative volatility processes in the modelling of options pricing.

Post-Hull-White, a series of stochastic models came into existence, but few of them managed to retain the attention of practitioners and researchers. The stochastic models of Heston (1993) and Heston-Nandi (2000), Implied Binomial Trees model of Rubinstein (1994), Derman and Kani (1994), and Dupire (1994), ARCH models of Engle (1995), stochastic jump diffusion model of Bates (1996), DVF model of Dumas, Fleming and Whaley (1998) and affine jump-diffusion model of Duffie, Pan and Singleton (2000) are some models that managed to gain some popularity. These models all suffered from the common weakness that the estimation of model parameters characterising stochastic volatility is quite computationally intensive.

Thus, to provide a more focused approach, we tested the applicability of the benchmark BS model (Black & Scholes, 1973) and its stochastic counterpart, Hull and White (1987, 1988). Despite different assumptions, both models remain the most dominant models of their type amidst analytical tractability. To evaluate the stability/robustness of these models during the most dynamic and turbulent financial changes, the models have been put through a complete cycle of financial swings. Furthermore, the models are passed through the data collected from the most steady-unsteady period of Indian financial frames. This phase in particular shows an extreme amount of unpredictability and thus provides the most apt situation for testing the relative competitiveness of the models. As the models will consider the extreme range of market (index) movements, the real time applicability of models becomes more feasible.

As all of the parameters of the BS model except volatility are directly observable from the market, the model's performance largely depends on the quality of volatility. Though there are several methods to measure volatility, only few of them are dominant. Generally, practitioners measure volatility in two ways: looking backward and looking forward. The implied volatility (IV), obtained from the market option prices, has been found to be forward looking. As IV incorporates market information embedded in options prices, it reflects the future volatility of the underlying asset and is thus very popular (Day & Craig, 1992; Edey & Elliot, 1992; Canina & Figlewski, 1993; Christensen & Prabhala, 1998; Ederington & Guan, 2002). We therefore utilised IV as an input in BS to determine the price of Nifty index options.

To justify a study pertaining to Nifty index options of India, we have compared and contrasted the log normal distribution of Nifty with its global counterparts such as FTSE 100, KOSPI, Nikkei 225, TAIEX, and RUSELL 2000. Figure 1 provides strong support to the question of "why India?" Figure 1 clearly exhibits that log normal frequency distribution of Nifty is most unique as it has the longest tails on both sides (for sample period January 2000–September 2013). Table 1 also clearly exhibits that, when comparing the six indices, the return distribution of Nifty is most unique and depicts the highest value of Kurtosis and Jarque-Bera (test of non-log-normality). The same pattern is also observed for the sample in this study (2006–2011). This finding implies that during the period of study, investors had great opportunity for extreme positive and negative returns while trading at Nifty. Furthermore, the positive value of skewness of Nifty also qualifies it for the purpose of this study. Other than Nifty, the skewness of all other global indices is negative (Table 1). Kurtosis of the Nifty supports the conclusion that, during the period 2006–2011, the probability of occurrence of extreme returns was more likely for Nifty compared to its global counterparts. At the same time, the probability of scenarios of extremely negative returns was not as likely. Accordingly, this research paper mainly focuses on inventing and determining the best alternative option pricing model that can define the right distributional assumptions for pricing S&P CNX Nifty 50 index option of India.

In addition to the distribution characteristics, the growing literature on option prices and exponential growth of Nifty index options on the bourse of National Stock Exchange (NSE) of India also motivated us to investigate the inter-competence of the two models in the Indian context. The only issue that needs to be managed effectively is the calibration of the model parameters. To determine the parameters of models and make these models consistent with market prices, this research paper has utilised the method of optimisation. The remainder of the paper is structured as follows: The section entitled 'Option Pricing Process' details the basic assumptions and properties of the BS and Hull and White pricing processes. Section 'Data Description' explores the data screening procedure of Nifty index options and briefly reviews the parameters estimation methods. Section 'Out-Of-Sample Pricing Performance' discuses the empirical results and critically examines the relative perfection of BS and HW. In addition, this section also briefly reviews the hedging effectiveness and correlation sensitivity of Hull and White models. The last section finally concludes the study.

Table 1
Statistics of frequency distribution of global indices

	FTSE_100	KOSPI	NIFTY	NIKKEI_225	TAIEX	RUSELL_ 2000
January 2000–September 2013						
Skewness	–0.14	–0.54	**–0.28**	–0.42	–0.17	–0.28
Kurtosis	8.88	8.19	**10.58**	9.26	5.25	7.2
Jarque-Bera	5006.69	3988.73	**8272.25**	5620.24	742.25	2586.47
Observations	3,473	3,402	3,434	3,378	3,433	3,456
January 2006–December 2011						
Skewness	–0.11	–0.56	**0.01**	–0.53	–0.38	–0.3
Kurtosis	9.35	9.46	**10.06**	11.46	5.43	6.82
Jarque-Bera	2545.47	2713.91	**3146.4**	4579.6	406.97	941
Observations	1,513	1,513	1,513	1,513	1,513	1,513

Financial Characteristics of Nifty

Figures 1–4 display various financial characteristics of Nifty index options that are vital for the applicability of option pricing models and also for the pricing of index options underlying the Nifty index. Figure 1 shows that, for the period of study, the frequency plot of Nifty index return is non-lognormal. Researchers rooted smile led prices bias of BS to this non-log normality of assets prices. Figure 2 provides evidence that the Nifty index return, and its implied volatility, are strongly negatively correlated, whereas Figure 3 shows that the asset return volatility tends to imply a mean reverting stochastic volatility process. Together, all three financial characteristics create the view that parameters of the HW stochastic volatility process (estimated from option prices) can be used to produce reliable predictions of the day-ahead relationship between Nifty index option prices and its index levels. The smile pattern exhibited in Figure 4 reveals the existence of unique implied volatilities for different sets of maturity and strike. Consequently, accurate pricing and hedging of options become typical tasks to achieve within the standard BS framework. However, in the stochastic framework of Hull and White, achieving accurate pricing and hedging is more challenging.

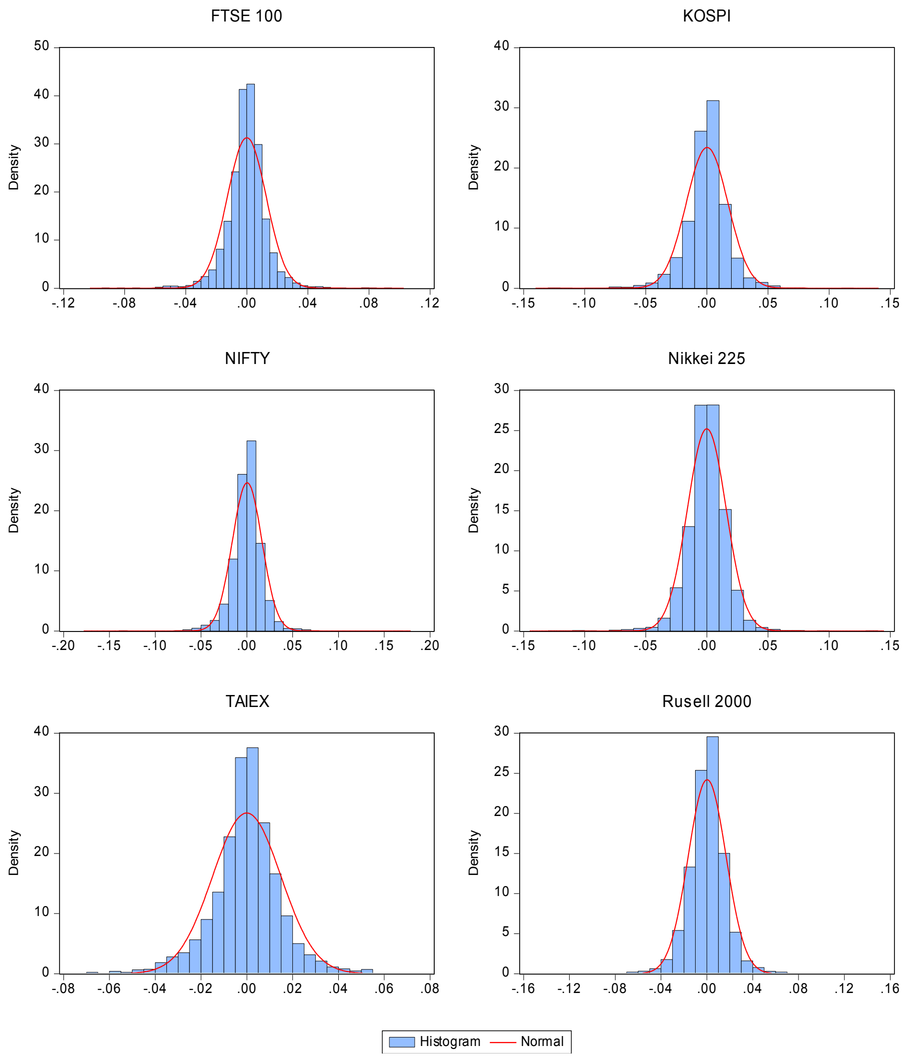

Figure 1. Log non-normal distribution of return of Nifty and its global counterparts (January 2000–September 2013)

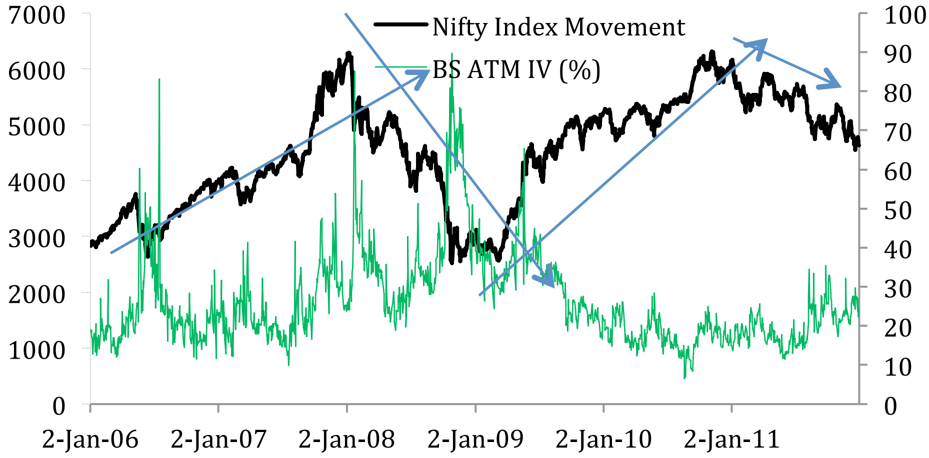

Figure 2. Leverage effect of Nifty index

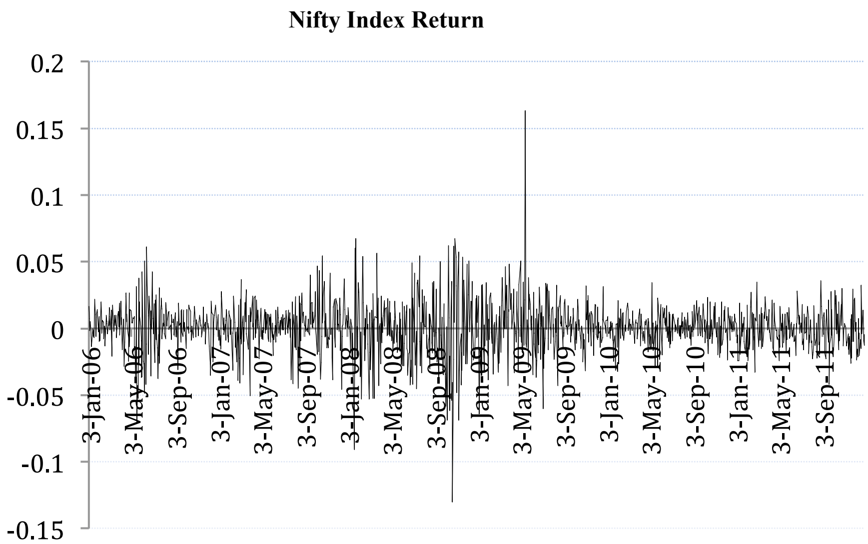

Figure 3. Volatility clustering of Nifty index return

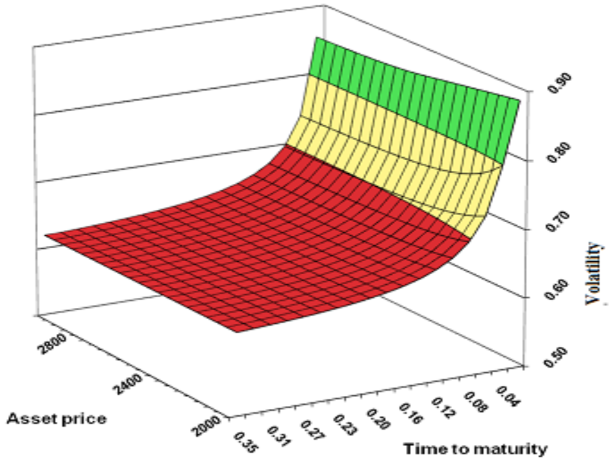

Figure 4. 3-D volatility smile surface of Nifty index options, observed on 24 October 2008 (Parameters: S = 2584, K = 2000 : 3000, r = 7.18%, q = 0, T = 5 : 90 Days)

OPTION PRICING PROCESS

Black–Scholes–Merton

The 'No Arbitrage Argument' is one of several generic approaches to asset pricing. This approach is also the essence of the benchmark BS partial differential equation (PDE), which can be solved numerically for various asset classes, even with special cases.

Black, Scholes and Merton assumed that the asset price follows a geometric Brownian process (Karatzas & Shreve, 1991) driven by a source of randomness, W_t:

$$dS = \mu S dt + \sigma S dW_t$$

Where μ is the expected rate of return also known as drift rate (in the BS framework it is same for all investors) and σ is the volatility of asset returns—both assumed to be constant. Their path breaking formula for pricing European call option is:

$$C_{BSM} = S.N(d_1) - X.e^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S/X) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

S is the current stock price, X is the option's strike price, r is the continuously compounded risk-free interest rate, T is the time to maturity in years, σ is the standard deviation of the price of the underlying stock, and $N(d)$ is the Gaussian distribution function.

Implied volatility

This method is the reverse approach to finding volatility. Instead of specifying a mathematically complex model and estimating its parameters, this method matches the cross-section of option prices with Black-Scholes model prices and calculates the volatility independent of the model parameters. Accordingly, this volatility is also known as model-free implied volatility. For the calculation of implied volatility, the choice of moneyness is extremely important. For example, the implied volatility extracted from the market data set often exhibits a smile. To adjust for this, researchers used at-the-money implied volatility for forecasting the prices of financial assets. Although this method alleviates the "smile" problem to some extent, it also discards the usage of all potential information contained in the rest of the option prices. To circumvent this outcome, we banked on parametric implied volatilities and extracted the implied volatility from a set of option prices, thereby ensuring the incorporation of all the information embedded in the option prices across moneyness and maturities. These volatilities are further utilised to forecast future expectations of the market participants; hence, this approach constitutes a forward-looking estimate of the volatility of the underlying asset. The calibration procedure of the implied volatility is discussed in section entitled Data Description–Calibration of Competing Models of this paper.

Hull-White (1987)

As stated earlier, to fulfil the empirical deficiencies of Black-Scholes, Hull and White (1987) developed a stochastic model. They assumed that the asset price and the instantaneous variance follow the following stochastic process in a risk-neutral world:

$$dS = bS_t dt + \sigma S_t dz$$

$$dV = \alpha V + \xi V dw$$

Where, b and α are the drifts of the asset price and variance respectively; V is the variance i.e. $V = \sigma^2$, ξ is the volatility of the variance; dz and dw are independent Wiener processes, therefore meaning that the asset price and volatility are not correlated; and α and ξ are independent of S. The option pricing formula of Hull-White relies on the distribution of the average variance V of the asset price process over the life of the option defined by the stochastic integral

$$C_{HW87} = \int C(\bar{v}) h(\bar{v} / \sigma_t^2) d\bar{v}$$

Where $C(\bar{v})$ is the BS formula with its usual notations, and $\bar{v}$ is the mean variance over the lifetime of the option, mathematically defined as

$$\bar{V} = \frac{1}{T-t}\int_t^T \sigma^2(t)dt,$$

Similar to BS, HW (1987) found that the conditional distribution of the terminal asset price is also log-normally distributed. However, the risk-neutral dynamics of the volatility do not depend on the asset price S. The previous two expressions do not exist for cases in which asset price and volatility are correlated. As the formula of HW is independent of investors' risk preferences, it does not incorporate risk premium in the option-pricing model, defined as

$$C(S,v,t) = e^{-r(T-t)}\int C(S_T, v_T, T) f(S_T / S, v) dS_T$$

Where $f(S_T | S, v)$ is the conditional distribution of asset price (S_T) and variance at time t; $C(S_T, v_T, T)$ is the traditional payoff function defined as $\max\{S_T - K, 0\}$. Utilising Taylor's series (Taylor, 1986), they expanded the above model with expected values and derived the option pricing formula

$$C_{HW87} = BS(\sigma^2) + \frac{1}{2}\frac{\partial^2 C(v)}{\partial v^2}\left\{\frac{2\sigma^4(e^k - k - 1)}{k^2} - \sigma^4\right\} + \ldots\ldots$$

$$C_{HW87} = BS(\sigma^2) + \frac{1}{2}\frac{Se^{(\mu - r)\tau}\sqrt{\tau}n(d_1)(d_1 d_2 - 1)}{4\sigma^3}\left\{\frac{2\sigma^4(e^k - k - 1)}{k^2} - \sigma^4\right\} + \ldots\ldots$$

where $k = \xi^2(T-t)$, $\tau = (T-t)$

Hull and White (1988) Correlated Stochastic Volatility Model

To rectify the empirical discrepancies of their own model, Hull and White (1988) conceptualised a new framework for stochastic volatility options pricing. The new framework dealt with controlling the non-lognormal characteristics of assets returns. To improve on the prior framework, they offered a more flexible distributional structure and relaxed the zero correlation restriction of Hull and White (1987). However, as with the previous version, they assumed that the asset price and its volatility follow a square root stochastic volatility process, defined as

$$dS = S\mu\, dt + S\sqrt{V}\, dz$$

$$dV = (\alpha + \beta V)dt + \xi\sqrt{V}\, dw$$

Similar to Hull and White (1987), *dz* and *dw* here are Wiener processes with correlation ρ, and ξ is the instantaneous volatility of the volatility $\sqrt{V}$. As, in an absolute sense, volatility cannot be negative, the term $\alpha + \beta V$ ensures a positive instantaneous variance (V) between the asset return and volatility.

Again utilising the second-order Taylor series expansion, Hull-White developed a closed form approximation around a constant volatility specification ($\xi = 0$) and developed the closed-form approximation for pricing European call option under stochastic correlated framework, defined as

$$C_{HW88} \approx f_0 + f_1\xi + f_2\xi^2$$

Where f_0 is the benchmark BSM formula $f_0 = C(\overline{V})$.

The price bias added to the BS formula yields the stochastic-volatility adjusted call price. However, in cases where the variance is constant, i.e. $\xi = 0$, Hull and White (1988) will converge with the traditional BS.

DATA DESCRIPTION

Figure 1 implicitly shows that the period of 2006–2011 was a dynamic one for traders and investors. This timeframe tested not only the quality of all entities related to capital markets, but also the robustness of their financial mathematical models. In the midst of this time frame, Nifty peaked early but then felt the impacts of the global financial crisis. However, it soon rose steadily but did not reach its previous high. Since then, Nifty's growth has been sluggish. In the beginning of 2006, the Indian economy expanded at its fastest pace, but it later slowed down. Accordingly, we identified this specific period for our research to test the effectiveness of stochastic HW and deterministic BS models. This period provides the best laboratory conditions to gauge the effectiveness of appropriate models and to measure the pros and cons of options pricing. To identify the most appropriate model, we have tested the models using data from this timeframe with the assumption that the best model will hold during any type of trading scenario. To determine the empirical performance of the models, we collected the historical data of S&P CNX Nifty 50 Index option contracts for the period specified, i.e. data for 1487 trading days. We have also collated the interest rate data yield of "91 Day T-Bill" for the period specified, i.e. 1 January 2006 to 31 December 2011. The data set was collected manually from the official browsers of the National Stock Exchange (NSE) and the Reserve Bank of India (RBI). Data on option type, strike price, underlying index price, maturity date and risk free interest rate have been cleaned and merged.

Data Screening Procedure

To ensure that the raw data are ideal to test the conceptual framework of Black-Scholes, and Hull-White (1987, 1988), we passed the sample data through five exclusionary filters, applied in sequence. Accordingly, we have tried to remove the irregularities in the options data, considered to be sensitive for our pricing analysis. First, call option prices not satisfying the arbitrage-cum-lower boundary conditions $Max\left(0, S - X, S_t \; - X\, e^{-r(T-t)}\right) \leq C_{Market} \leq S$ were removed from the data set. Thereafter, illiquid and extremely sensitive options not satisfying the conditions such as number of trading contract/open interest equal to zero, number of traded contracts less than 50, maturity $T > 90$ and $T < 3$ days and moneyness $+15\% < \left(\frac{S}{X} - 1\right) \; and \; \left(\frac{S}{X} - 1\right) < -15\%$ were discarded from the data set. The above exclusionary filters resulted in the rejection of over 94% of the sample option data. Table 2 displays the descriptive summary statistics for the Nifty index option during the period specified.

Table 2
Filter statistics of Nifty index call options (2006–2011)

	Year						Sub Total
	2006	2007	2008	2009	2010	2011	2006–2011
Total Call Contracts	44,555	38,890	72,494	93,860	95,008	17,9695	52,4502
Criteria	**Data Rejected**						
No Trading Volume/Open Interest	35,304	29,052	56,835	75,785	76,514	157,009	430,499
No. of Traded Contracts ≤ 50	5,182	4,163	7,216	9,137	6,481	7,394	39,573
Moneyness > +15%	170	366	211	1,459	1,055	1,132	4,393
Moneyness < -15%	84	9	1,866	738	263	2,673	5,633
Maturity > 90 Days	0	2	543	390	1,533	2,029	4,497
Maturity < 3 Days	385	490	551	486	502	520	2,934
No Arbitrage Relationship	584	573	123	196	932	989	3,397
Rejected Data	41,709	34,655	67,345	88,191	87,280	171,746	490,926
Rejected Data (%)	93.61	89.11	92.90	93.96	91.87	95.58	93.60
Remaining Data	2,846	4,235	5149	5,669	7,728	7,949	33,576
Remaining Data (%)	6.39	10.89	7.10	6.04	8.13	4.42	6.40

Option Categories

To provide a tabular and sequential analysis, we framed moneyness in five categories and maturity in three categories. The moneyness *(S-X)/X* groups are categorised as:

$$\begin{pmatrix} DOTM \\ OTM \\ ATM \\ ITM \\ DITM \end{pmatrix} \quad if\ moneyness\left(S/_{K}-1\right) \begin{cases} \in[-15\%,-10\%) \\ \in[-10\%,-5\%) \\ \in[-5\%,+5\%] \\ \in(+5\%,+10\%] \\ \in(+10\%,+15\%] \end{cases}$$

Whereas maturity (T) is grouped as:

$$\begin{pmatrix} Short\,Term \\ Medium\,Term \\ Long\,Term \end{pmatrix} \quad if\; Time\; to\; Maturity\,(T) \begin{cases} \in \left[5, 30\right] \; Days \\ \in \left(30, 60\right] \; Days \\ \in \left(60, 90\right] \; Days \end{cases}$$

The combination of these two factors resulted in fifteen categories of moneyness-maturity. We then categorically placed the filtered data in this matrix composition. Table 3 displays the summary statistics of this matrix. The following abbreviations have been used throughout this paper: DOTM: deep-out-of-the-money, OTM: out-of-the-money, ATM: at-the-money, ITM: in-the-money, and DOTM: deep-in-the-money.

Table 3
Descriptive statistics of Nifty index call option for the year 2006-2011 (post filtration)

		Call Moneyness ((S/K)-1)					Total/Sub Total
		DOTM	OTM	ATM	ITM	DITM	
Maturity	Short	2,511	3,287	8,169	1,498	607	16,072
		7.48%	9.79%	24.33%	4.46%	1.81%	47.87%
	Medium	1,737	2,706	6,049	1,011	259	11,762
		5.17%	8.06%	18.02%	3.01%	0.77%	35.03%
	Long	869	1,681	2,939	210	43	5,742
		2.59%	5.01%	8.75%	0.63%	0.13%	17.10%
Total/Sub Total		5,117	7,674	17,157	2,719	909	33,576
		15.24%	22.86%	51.10%	8.10%	2.71%	100.00%

Performance Evaluation Methodology

To determine the relative competence and out-of-sample forecasting competiveness of the models, we juxtaposed HW and BS relative to the market. Furthermore, to ensure the quality of analysis, we employed the techniques of error metrics *viz.* Percentage Mean Error (MPE) and Mean Absolute Percentage Error (MAPE) to determine the parities of the models. The mathematical expressions of the two error metrics are

$$\text{Mean Percentage Error (MPE)} = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{\left(C_i^{Model} - C_i^{Market}\right)}{C_i^{Market}}\right]$$

$$\text{Mean Absolute Percentage Error (MPE)} = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|\left(C_i^{Model} - C_i^{Market}\right)\right|}{C_i^{Market}}$$

Where C_i^{Model} and C_i^{Market} is the expected and market price of the i^{th} observation, and n is the total number of observations. Positive (negative) MPE implies that the model overprices (under prices) specific options, while the value of MAPE helps determine whether the model provides a good approximation relative to the market.

Calibration of Competing Models

In a stochastic environment, the calibration of parameters is extremely cumbersome. However, studies reveal that the stable parameters can be deduced to the closest proximity by minimising the price bias of the model and the market, independent of stochastic and deterministic framework. This process is widely known as optimisation (Rubinstein, 1985; Rouah & Vainberg, 2007). The simplest generalised optimisation function is $f(\Omega)$

$$f(\Omega) = \min_{\Omega}\sum_{i=1}^{n}\left[C_{Model} - C_{Market}\right]^2$$

Where Ω is a set of vector parameters of models to be calibrated daily. The optimal set of parameters extracted from the previous day will be embedded in the models to price the current day options. This estimation procedure is repeated for each day of the sample data. The advantage of this method is that, in addition to providing stable parameters, it also incorporates information from the market (inherent in the historical data of underlying asset) in the option prices. However, compared to BS, calibration of Hull and White (1987, 1988) models is quite complex because, in the latter model, four parameters need to be estimated concurrently whereas in the former model, only one, i.e. implied volatility (IV), is required.

OUT-OF-SAMPLE PRICING PERFORMANCE

To analyse the competiveness of the classic Black-Scholes model versus stochastic Hull-White models (Hull and White, 1987, 1988), we examined the pricing correlations between the models. We thoroughly evaluated various combinations of moneyness and maturity depicting volatility, price and error statistics of BS and HW's. Tables 4, 5, 6 and 7 display descriptive statistics of these combinations. The outcomes of cross-sectional, comparative and analytical study of the given tables will decide how the models compare to one another. This section is intended (moneyness-maturity wise) to identify the best model in a particular category, based on the relative error performance.

Table 4 displays the dependence of implied volatility on maturity and moneyness. It clearly supports the empirical research work of Merton (1976 a, b), Scott (1987), Johnson and Shanno (1987), and Wiggins (1987) and finds that implied volatility varies systematically with respect to maturity and moneyness. Table 4 also shows that implied volatility tends to vary from DOTM to DITM options and makes a systematic upward trend when it deviates from ATM. However, the variation in implied volatility ranging from DOTM to DITM is highest in the case of BSM followed by HW87 and HW88. This variation depicts the models' volatility smile capturing capacity. Table 4 demonstrates that HW88 explains the smile phenomenon more profoundly.

Table 4
Implied Volatility Statistics of Black-Scholes & Hull-White's model

Models		DOTM	OTM	ATM	ITM	DITM	Total
				Moneyness Statistics			
BS IV	Average	0.21	0.20	0.19	0.21	0.25	0.20
	Std. Dev.	0.09	0.09	0.08	0.08	0.09	0.08
HW87	Average	0.21	0.20	0.19	0.21	0.26	0.21
	Std. Dev.	0.10	0.11	0.08	0.07	0.09	0.10
HW88	Average	0.20	0.20	0.18	0.21	0.24	0.20
	Std. Dev.	0.10	0.10	0.07	0.09	0.10	0.10
	No. of Observations	5117	7674	17157	2719	909	33576
				Moneyness-Maturity Statistics			
				Time to maturity (T ≤ 30)			
BS IV	Average	0.23	0.22	0.20	0.22	0.25	0.21
	Std. Dev.	0.10	0.09	0.08	0.08	0.10	0.09
HW87	Average	0.21	0.20	0.19	0.21	0.25	0.20
	Std. Dev.	0.13	0.11	0.11	0.11	0.12	0.12
HW88	Average	0.20	0.20	0.19	0.20	0.24	0.20
	Std. Dev.	0.11	0.11	0.11	0.14	0.17	0.13
	No. of Observations	2511	3287	8169	1498	607	16072

(continued on next page)

Table 4 *(continued)*

Models		DOTM	OTM	ATM	ITM	DITM	Total
		Time to maturity $(30 < T \leq 60)$					
BS IV	Average	0.22	0.20	0.19	0.21	0.26	0.20
	Std. Dev.	0.10	0.09	0.08	0.08	0.09	0.08
HW87	Average	0.22	0.20	0.18	0.20	0.25	0.20
	Std. Dev.	0.15	0.14	0.14	0.14	0.13	0.14
HW88	Average	0.22	0.19	0.18	0.20	0.25	0.21
	Std. Dev.	0.13	0.12	0.13	0.09	0.12	0.11
	No. of Observations	1737	2706	6049	1011	259	11762
		Time to maturity $(60 < T \leq 90)$					
BS IV	Average	0.17	0.16	0.16	0.17	0.20	0.16
	Std. Dev.	0.06	0.06	0.05	0.06	0.06	0.06
HW87	Average	0.08	0.15	0.14	0.16	0.19	0.15
	Std. Dev.	0.16	0.05	0.06	0.05	0.05	0.08
HW88	Average	0.16	0.15	0.14	0.16	0.19	0.15
	Std. Dev.	0.08	0.05	0.08	0.05	0.05	0.08
	No. of Observations	869	1681	2939	210	43	5742

Table 5 shows the price statistics of S&P CNX Nifty Index call options (moneyness-maturity) for the time period ranging from 1 January 2006 to 31 December 2011. It shows that the values of call options depend on moneyness and maturity. The price pattern exhibited in Table 5 validates the theory of option prices as the maturity-moneyness sequences of call options follow the ascending order understood by the series: DOTM < OTM < ATM < ITM < DITM and short term < medium term < long term.

Table 5
Price statistics of Black-Scholes & Hull-White's models

Models		DOTM	OTM	ATM	ITM	DITM	Total
				Moneyness Statistics			
Market	Average	15.16	38.37	145.49	377.23	547.62	130.80
	Std. Dev.	21.92	37.52	88.51	94.85	124.88	138.82
BS	Average	15.63	39.59	144.02	372.87	539.50	129.82
	Std. Dev.	25.13	40.33	88.47	93.79	124.25	137.39
HW87	Average	15.61	38.79	148.04	369.63	546.80	125.79
	Std. Dev.	22.65	45.00	78.46	88.77	116.25	126.23
HW88	Average	14.44	35.98	142.37	372.52	543.63	128.76
	Std. Dev.	20.90	43.43	76.81	82.38	115.49	122.20
	No. of Observations	5117	7674	17157	2719	909	33576
				Moneyness-Maturity Statistics			
				Time to maturity (T ≤ 30)			
Market	Average	6.17	17.43	106.54	352.24	539.23	111.88
	Std. Dev.	10.12	22.14	76.78	89.65	121.37	142.42
BS	Average	5.08	16.12	103.43	346.28	530.58	108.98
	Std. Dev.	11.17	22.56	74.99	87.53	119.79	140.23
HW87	Average	6.32	18.08	109.45	343.89	530.11	109.35
	Std. Dev.	11.62	26.08	80.12	88.21	119.02	139.01
HW88	Average	6.76	17.34	108.19	348.22	533.92	111.86
	Std. Dev.	10.89	25.22	80.67	86.45	117.67	137.98
	No. of Observations	2511	3287	8169	1498	607	16072

(continued on next page)

Table 5 *(continued)*

Models		DOTM	OTM	ATM	ITM	DITM	Total
Time to maturity (30 < T ≤ 60)							
Market	Average	21.89	47.73	171.06	396.51	549.19	148.36
	Std. Dev.	26.60	39.00	83.72	86.54	121.47	137.17
BS	Average	23.21	50.09	170.14	393.92	542.99	148.27
	Std. Dev.	30.63	41.42	83.09	85.30	122.69	135.76
HW87	Average	24.08	49.98	168.67	398.67	545.12	148.04
	Std. Dev.	33.46	45.59	76.84	82.79	102.31	128.20
HW88	Average	22.62	46.93	171.02	396.56	547.38	146.02
	Std. Dev.	32.21	44.69	76.31	82.01	99.07	124.26
	No. of Observations	1,737	2,706	6,049	1,011	259	11,762
Time to maturity (60 < T ≤ 90)							
Market	Average	27.68	64.27	201.10	462.61	656.64	147.78
	Std. Dev.	25.29	36.76	79.17	98.17	144.09	124.29
BS	Average	30.96	68.55	203.06	461.26	644.49	150.38
	Std. Dev.	28.50	39.68	79.93	96.46	147.39	123.51
HW87	Average	29.21	68.51	206.79	467.73	641.36	149.09
	Std. Dev.	22.87	33.32	78.43	88.32	147.42	122.47
HW88	Average	26.02	62.76	202.67	464.49	646.67	148.97
	Std. Dev.	20.61	36.39	73.45	83.67	151.73	118.37
	No. of Observations	869	1,681	2,939	210	43	5,742

Table 6 demonstrates the price effectiveness of the models underlying their cross-sectional and comparative Mean Percentage Error (MPE) analysis across moneyness and maturity groups.

Table 6
Mean percentage price bias statistics of Black-Scholes & Hull-White's models

Models		DOTM	OTM	ATM	ITM	DITM	Total
Moneyness Statistics							
BS	Average	–0.28	–0.07	–0.01	–0.01	–0.02	–0.06
	Std. Dev.	0.61	0.45	0.17	0.04	0.02	0.35
HW87	Average	–0.25	–0.05	0.03	0.04	–0.01	–0.04
	Std. Dev.	0.67	0.44	0.19	0.03	0.04	0.32
HW88	Average	–0.23	–0.04	0.03	0.03	–0.03	–0.03
	Std. Dev.	0.69	0.41	0.19	0.04	0.03	0.30
	No. of Observations	5,117	7,674	17,157	2,719	909	33,576
Moneyness-Maturity Statistics							
Time to maturity (T ≤ 30)							
BS	Average	–0.56	–0.24	–0.03	–0.02	–0.02	–0.15
	Std. Dev.	0.61	0.57	0.22	0.03	0.02	0.43
HW87	Average	–0.49	–0.23	–0.03	–0.03	–0.04	–0.13
	Std. Dev.	0.57	0.49	0.28	0.06	0.04	0.39
HW88	Average	–0.47	–0.21	0.02	0.02	–0.03	–0.11
	Std. Dev.	0.54	0.46	0.26	0.04	0.07	0.34
	No. of Observations	2,511	3,287	8,169	1,498	607	16,072
Time to maturity (30 < T ≤ 60)							
BS	Average	–0.06	0.06	0.00	-0.01	-0.01	0.00
	Std. Dev.	0.46	0.29	0.09	0.04	0.03	0.24
HW87	Average	–0.05	0.05	0.02	0.03	0.04	0.05
	Std. Dev.	0.48	0.24	0.05	0.05	0.05	0.27
HW88	Average	–0.03	0.04	0.01	0.02	0.03	0.04
	Std. Dev.	0.46	0.24	0.05	0.06	0.05	0.24
	No. of Observations	1,737	2,706	6,049	1,011	259	11,762

(continued on next page)

Table 6 *(continued)*

Models		DOTM	OTM	ATM	ITM	DITM	Total
			Time to maturity (60 < T ≤ 90)				
BS	Average	0.12	0.08	0.01	0.00	–0.02	0.05
	Std. Dev.	0.42	0.23	0.09	0.04	0.04	0.22
HW87	Average	0.10	0.09	0.03	0.04	–0.04	0.04
	Std. Dev.	0.41	0.22	0.10	0.04	0.05	0.21
HW88	Average	0.07	0.05	0.02	–0.02	–0.04	0.02
	Std. Dev.	0.48	0.26	0.08	0.05	0.06	0.24
	No. of Observations	869	1,681	2,939	210	43	5,742

Based on the data presented in Table 6, it can be concluded that the model of Black-Scholes-Merton and Hull-White (1987) overprice DOTM, OTM, ATM, ITM options and underprice DITM Nifty index call options, both in terms of moneyness. Table 6 clearly shows that the BS model severely mispriced DOTM & OTM, while Hull and White (1987, 1988) were relatively better at pricing these options. In all three models, we noticed a systematic decrease in the price error going from DOTM to DITM. The sequence is in the following pattern:

$$\textit{Moneyness}\begin{cases}\text{BSM : ITM (1\%)} \leq \text{ATM (-1\%)} < \text{DITM (-2\%)} < \text{OTM (-7\%)} < \text{DOTM (-28\%)} \\ \text{HW87 : DITM (-1\%)} < \text{ATM (+3\%)} < \text{ITM (+4\%)} < \text{OTM (-5\%)} < \text{DOTM (-25\%)} \\ \text{HW88 : DITM (-3\%)} < \text{ITM (+3\%)} \leq \text{ATM (+3\%)} < \text{OTM (-4\%)} < \text{DOTM (-23\%)}\end{cases}$$

Analysing the three models together, we concluded that the price variation across moneyness was lowest in the case of the Hull-White (1988) model compared to the other two models. When Tables 4, 5 and 6 were examined jointly, we concluded that, of the three models that have been discussed, the capability of the correlated version of HW model to explaining/capturing the volatility smile is higher than the uncorrelated HW and classical BS. We also identified that, among all of the models, the price error of HW88 is the lowest. It prices DOTM & OTM call options better than the other two with the pricing error being close to –23%. However, Hull-White (1987) differs marginally with a pricing error close to –25%. Table 6 reveals that the pricing error of Hull-White (1988) is lowest, while the pricing error of BS is the highest in analysing Nifty index options.

Categorically, we find that BS and HW (1987) underpriced short-term options of all moneyness, while the stochastic variant of HW, i.e. HW (1988), overpriced ATM and ITM options. Table 6 shows that all models tend to overprice long-term DOTM, OTM and ATM options and underprice DITM options. Table 5 also presents evidence that, of the three models, the price error of HW (1988) is lowest across moneyness and thus it is the best to price Nifty call index options. Pricing performance of BS is also comparable, to a great extent, at least for ATM, ITM and DITM options. However, its performance in DOTM and OTM categories is extremely poor as it severely underpriced short-term DOTM & OTM options. The short-term pricing behaviour sequence of the three models can be arranged in the following pattern:

$$
\textit{ShortTerm}\begin{cases}\text{BSM: DITM (-2\%)} \le \text{ITM (-2\%)} < \text{ATM (-3\%)} < \text{OTM (-24\%)} < \text{DOTM (-56\%)}\\ \text{HW87: ITM (-3\%)} \le \text{ATM (-3\%)} < \text{DITM (-4\%)} < \text{OTM (-23\%)} < \text{DOTM (-49\%)}\\ \text{HW88: ITM (+2\%)} \le \text{ATM (+2\%)} < \text{DITM (-3\%)} < \text{OTM (-21\%)} < \text{DOTM (-47\%)}\end{cases}
$$

Similarly, medium and long-term pricing sequence can be arranged like:

$$
\textit{Med. Term}\begin{cases}\text{BSM: ATM (0\%)} < \text{DITM (-1\%)} \le \text{ITM (-1\%)} < \text{OTM (+6\%)} < \text{DOTM (-6\%)}\\ \text{HW87: ATM (+2\%)} < \text{ITM (+3\%)} < \text{DITM (+4\%)} < \text{DOTM (−5\%)} < \text{OTM (+5\%)}\\ \text{HW88: ATM (+1\%)} < \text{ITM (+3\%)} < \text{DITM (+3\%)} < \text{OTM (+4\%)} < \text{DOTM (-3\%)}\end{cases}
$$

$$
\textit{LongTerm}\begin{cases}\text{BSM: ITM (0\%)} < \text{ATM (+1\%)} < \text{DITM (-2\%)} < \text{OTM (+8\%)} < \text{DOTM (+12\%)}\\ \text{HW87: ATM (+3\%)} < \text{DITM (-4\%)} < \text{ITM (+4\%)} < \text{OTM (+9\%)} < \text{DOTM (+10\%)}\\ \text{HW88: ITM (-2\%)} < \text{ATM (+2\%)} < \text{DITM (-4\%)} < \text{OTM (+5\%)} < \text{DOTM (+7\%)}\end{cases}
$$

A clear pattern results in short-term maturity options, but in medium and long-term categories the HW models cause overpricing of OTM and ATM options but underpricing of DITM Nifty index call options. Hence, we observe a systematic reduction (though not definite) in the price error of models in the following sequence: short term > medium term > long term. This finding implies that, with the increase in maturity, the performance of the models deteriorates. The data in Table 7, which exhibits the absolute percentage bias of models, do not reveal any new information but re-validate and support the results of Table 6. Furthermore, the pattern of the sequential effect indicates that the time to maturity is a crucial factor in the pricing performance of the discussed models, and the sequential representation of the models remains unchanged in all maturity classes (the price error decreases from DOTM to DITM options). Overall, Tables 6 and 7 jointly show that the pricing performance of HW (1988) is superior in options that are usually heavily traded *viz.* DOTM, OTM & ATM (Table 3). In addition to tables 6 and 7, figures 5 and 6 also provide additional information to support the result that the pricing bias of the correlated HW model is lower than the uncorrelated HW and benchmark BS model in DOTM, OTM & ATM groups. Figures 5 and 6 show the absolute and relative pricing bias of Black-Scholes and Hull-White's (1987, 1988) models relative to the market.

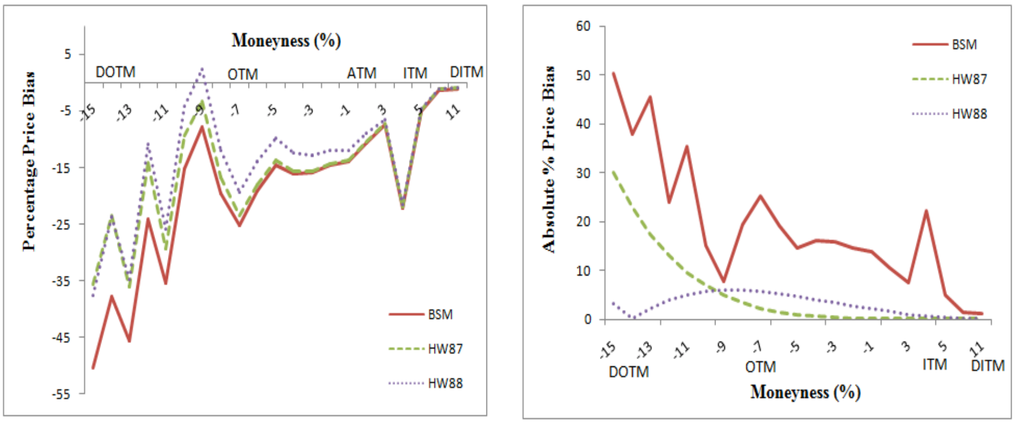

Figure 5. Price bias of Black-Scholes and Hull-White's (1987, 1988) models

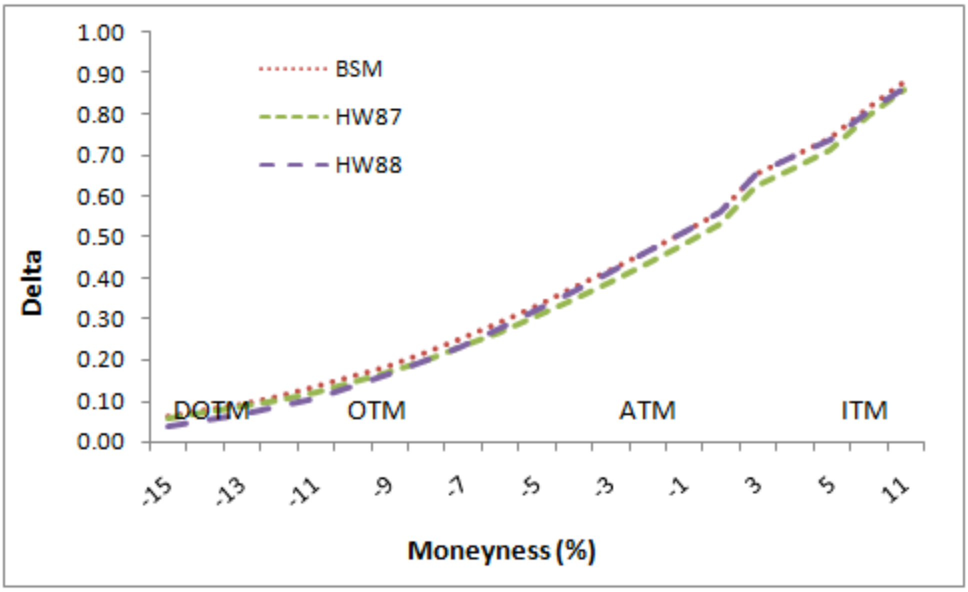

Figure 6. Hedging performance of Black-Scholes and Hull-White (1987, 1988)
(refer Table 4)

Table 7
Mean absolute percentage price bias statistics of Black-Scholes & Hull-White's model

Models		DOTM	OTM	ATM	ITM	DITM	Total
				Moneyness Statistics			
BS	Average	0.52	0.32	0.10	0.03	0.02	0.20
	Std. Dev.	0.42	0.33	0.14	0.03	0.02	0.30
HW87	Average	0.54	0.32	0.10	0.04	0.02	0.21
	Std. Dev.	0.45	0.35	0.13	0.02	0.03	0.35
HW88	Average	0.51	0.33	0.09	0.03	0.03	0.20
	Std. Dev.	0.44	0.36	0.12	0.05	0.02	0.34
	No. of Observations	5117	7674	17,157	2719	909	33,576
				Moneyness-Maturity Statistics			
				Time to maturity (T ≤ 30)			
BS	Average	0.70	0.48	0.13	0.03	0.02	0.28
	Std. Dev.	0.45	0.39	0.18	0.03	0.02	0.37
HW87	Average	0.69	0.42	0.12	0.04	0.03	0.26
	Std. Dev.	0.46	0.36	0.16	0.05	0.04	0.32
HW88	Average	0.68	0.43	0.11	0.04	0.02	0.28
	Std. Dev.	0.45	0.34	0.11	0.06	0.04	0.31
	No. of Observations	2511	3287	8169	1498	607	16,072
				Time to maturity (30 < T ≤ 60)			
BS	Average	0.36	0.21	0.06	0.03	0.02	0.14
	Std. Dev.	0.29	0.20	0.07	0.03	0.02	0.19
HW87	Average	0.34	0.23	0.03	0.07	0.04	0.12
	Std. Dev.	0.21	0.24	0.03	0.06	0.04	0.19
HW88	Average	0.37	0.21	0.03	0.05	0.04	0.11
	Std. Dev.	0.19	0.22	0.03	0.05	0.02	0.18
	No. of Observations	1737	2706	6049	1011	259	11,762

(continued on next page)

Table 7 *(continued)*

Models		DOTM	OTM	ATM	ITM	DITM	Total
		Time to maturity $(60 < T \leq 90)$					
BS	Average	0.29	0.16	0.06	0.03	0.03	0.12
	Std. Dev.	0.32	0.18	0.06	0.03	0.03	0.19
HW87	Average	0.26	0.15	0.05	0.05	0.04	0.11
	Std. Dev.	0.36	0.14	0.07	0.03	0.04	0.17
HW88	Average	0.24	0.11	0.05	0.03	0.04	0.09
	Std. Dev.	0.37	0.13	0.08	0.04	0.04	0.17
	No. of Observations	869	1681	2939	210	43	5742

The combined analysis of Tables 5 and 6 demonstrates that the incorporation of stochastic volatility leads to a higher pricing effectiveness, but not across all groups of maturity and moneyness. As estimation of stable parameters in a stochastic framework is difficult, traders and practitioners may not prefer to switch to this complex mode of option pricing. Thus, we conclude that the performance of models varies based on how the models incorporate the financial characteristics of various observable and non-observable parameters. Furthermore, to ensure overall applicability of Black-Scholes and Hull-White models and to provide the most apt model to traders for pricing options, cross-sectional empirical analysis of the same data needs to be performed with other models of the family in the desired period.

Hedging Effectiveness

Table 8 reveals that the delta hedge performance of the Black-Scholes model is volatile compared to the Hull-White (1987, 1988) model. This finding indicates that the Hull-White models follow a return distribution. Overall, the performance of the Hull and White (1988) model is better than that of the other two models, perhaps because of the symmetric distribution of the Nifty index returns, especially during the period of the hedge. Delta, the hedging parameter of the models, is computed using the following parameters: index/underlying price 4214, risk free rate of interest 9.6%, initial volatility (volatility of index return) 35.44%, long run volatility 20%, half-life to volatility shocks 3 years, volatility of volatility 42.23%, correlation of asset return and volatility –0.64, time to maturity 28 days, and exercise price ranging 4600 to 6100 (in multiples of 100). Figure 6 graphically displays the hedging performance of the models in question.

Table 8
Hedging effectiveness: Price and delta statistics/simulation of Nifty index option

Moneyness (%)	Market Price	Models Price			Delta of Models		
		BS	HW87	HW88	BS	HW87	HW88
10.89	454.6	449.61	450.43	450.09	0.8797	0.8592	0.8630
8.05	365.45	360.69	361.43	361.60	0.8183	0.7937	0.8063
5.35	294.05	279.26	279.79	280.74	0.7424	0.7143	0.7355
4.05	311	242.12	242.54	243.92	0.7000	0.6704	0.6952
2.78	224.35	207.74	208.04	209.84	0.6552	0.6245	0.6520
0.33	165.5	147.94	148.10	150.56	0.5611	0.5294	0.5589
–0.85	142.45	122.70	122.85	125.50	0.5133	0.4816	0.5102
–2.00	117.65	100.57	100.74	103.48	0.4658	0.4345	0.4610
–3.13	96.8	81.43	81.67	84.38	0.4193	0.3889	0.4120
–4.23	77.65	65.14	65.46	68.05	0.3744	0.3452	0.3641
–5.30	60.2	51.46	51.90	54.27	0.3317	0.3040	0.3178
–6.36	49.7	40.15	40.71	42.80	0.2915	0.2655	0.2740
–7.38	41.35	30.94	31.62	33.39	0.2541	0.2300	0.2330
–8.39	29.25	23.55	24.32	25.76	0.2198	0.1977	0.1955
–9.38	19.2	17.70	18.55	19.66	0.1886	0.1686	0.1616
–10.34	15.5	13.14	14.04	14.84	0.1606	0.1427	0.1315
–11.28	14.95	9.64	10.56	11.09	0.1357	0.1198	0.1054
–12.21	9.2	6.99	7.90	8.20	0.1138	0.0999	0.0829
–13.11	9.2	5.01	5.88	6.00	0.0947	0.0826	0.0641
–14.00	5.7	3.54	4.36	4.35	0.0783	0.0679	0.0486
–14.87	5	2.48	3.22	3.12	0.0642	0.0554	0.0360

Correlation Sensitivity of Hull and White (1988)

Figure 7 shows the correlation sensitivity of Hull-White (1988) with respect to the market price and indicates that the degree of correlation has a significant impact on the option prices. The price bias of a model and the market is the lowest in the case of a negative correlation, followed by zero correlation, and then positive correlation. This finding supports the fact that Nifty return and implied volatility follow a negative correlation (Figure 2) and indicates that negative correlation invariably drives down the OTM call option prices, whereas positive correlation drives them up.

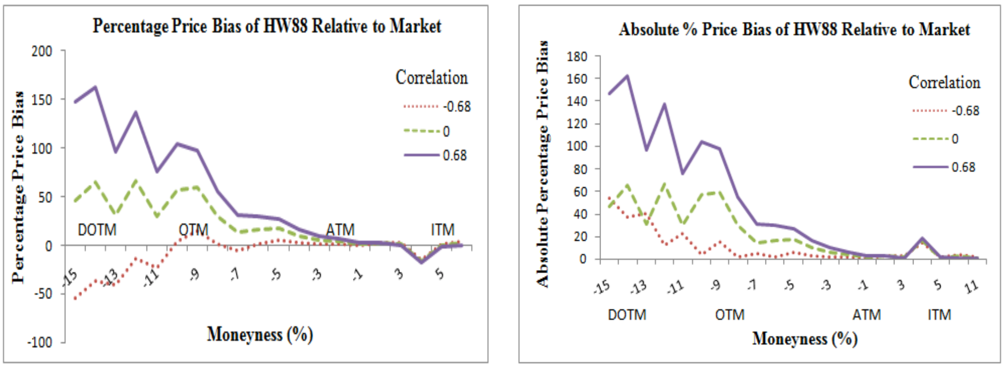

Figure 7. Correlation sensitivity of Hull-White (1988)

Although the Hull-White (1988) model suffers from some serious limitations such as a non-robust approximation and limits on the values of parameters which it accepts (because of Taylor series), the model is used extensively for pricing various option instruments. Such instruments include interest rate options, path dependent options, future options and exotic options, due to its adaptability and flexibility in estimation of parameters and its capacity to deduce the model parameters from discrete market data.

CONCLUSION

The Hull-White stochastic volatility models have been derived from empirical research analysis, which demonstrates that the assets return distribution is non-lognormal (Mandelbrot, 1963; Fama, 1965). The model wields its effect in strong moneyness and maturity pricing biases of Black-Scholes (Smith, 1976; Rubinstein, 1985; Wiggins, 1987; Derman & Kani, 1994). Having considered the concept and its applicability, we focus on determining the best alternative model based on specific distributional assumptions. We acknowledge that the core platform of almost all the stochastic models was substantially indicating toward the flexible distributional structure, which not only correlated underlying stock returns and its volatility, but also controlled the level of skewness and kurtosis. We deduced that the stochastic models improve pricing error significantly when compared to the classical BS model. Among the various options available, this paper finds the most suitable model and ensures that it works with actual data and outperforms other competitive models. We determined that the Hull-White model met these criteria. We reject the claim that HW's model outperforms the BS because the former is unable to remove the pricing bias completely. Unknown factors such as random jump, market forces and other various uncontrollable dynamics may still cause options price volatility. As the Hull-White model is adopted as the most successful model, it can be assured that this model will perform even better when utilised in normal, average and stable conditions with better clutched controllability. The HW model satisfies the desire to keep investments protected under normal conditions. Following this analysis, we identify a dominant model that remained successful through one of the most difficult phases of the Indian economy.

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