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Spatial Interaction Model and Characteristic Measure Based on Master Equation: Case Studies with Beijing-Tianjin-Hebei Population Flow

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Abstract: This study establishes a spatial interaction model based on master equation to address the problems of lack of quantitative representation and visual analysis capability in the existing master equation approach to spatial interaction models. provides a quantitative description of regional interaction strength. Taking the Beijing-Tianjin-Hebei region as an example, the master equation-based spatial interaction model is verified to have the ability to explore the spatial development pattern of cities, reveal the dynamic development mechanism and influencing factors of cities, and explore more realistic urban development trends through population flow data. The results show that: (1) the spatial interaction network of cities in the Beijing-Tianjin-Hebei region shows an integrated development network pattern with Beijing as the core of the radial network, supplemented by the network of small central cities, and the imbalance of inter-city development is still a problem for the realization of synergistic development in the Beijing-Tianjin-Hebei region. The imbalance of intercity development is still a problem for the Beijing-Tianjin-Hebei region to achieve synergistic development. (2) Compared with the classical gravity model, the main equation spatial interaction model simulates the intensity of the urban system from the perspective of openness and randomness, and shows a better performance in the details of the network structure, showing a "core with many small canters" of the circle development trend.

Keywords: Beijing-Tianjin-Hebei; Gravity Model; Interaction; Master Equation;

1.0 Introduction

Cities and urban agglomerations, as territorial complexes of human and natural elements, are complex open systems with stochastic, dynamic changes. Dynamism is the result of a series of parallel, unrelated or interrelated, individual or collective choices within the urban system. Uncertainty in the decision-making process disrupts the evolutionary path of the urban system, which in turn leads to the stochastic nature of the urban system. In the context of the big data era, the emergence of ubiquitous data has enabled the study of urban spatial interactions to break through the traditional confines of individual closed cities and begin to shift to an open urban cluster perspective. This shift, on the one hand, increases the complexity of calculating urban spatial interactions; on the other hand, it reflects the transformation of cities or city clusters from closed to open and from static to dynamic. However, most existing methods for calculating spatial interactions among cities rely on yearbook or census data, which are deficient in timeliness, continuity and directionality (Wang et al., 2021).

As an important theoretical basis for regional economic linkages, spatial interactions are the theoretical basis for regional development integration. Scholars from various fields have also proposed and developed a series of spatial interaction models in economy, tourism, transportation, urban planning, etc (Sobkowicz et al., 2013; Stollenwerk & Briggs, 2000; Zhao et al., 2018).The distribution of frequent, bi-directional or multi-directional flows in terms of intensity and direction in urban agglomerations can reflect the characteristics of urban functional linkages is now often applied to the simulation of spatial interactions in urban agglomerations(Xuan et al., 2016; Lee et al., 2011) .Currently, the mainstream spatial interaction models include gravity model, maximum entropy model and Markov model. The gravitational model, although extremely analogous, suffers from two drawbacks, namely the empiricism (Olsson, 1967) and the existence of a break-point paradox (Li et al., 2012).

The maximum entropy model divides spatial interactions into two levels: macroscopic and microscopic, and the macroscopic state with the most microscopic states is the most likely distribution of the system. Compared with the gravity model, the maximum entropy model draws on the ideas of statistical mechanics and has a solid theoretical foundation, and the damping function in exponential form circumvents the break-point paradox. However, the maximum entropy model assumes that the system is closed, while the real urban cluster is an open and complex system. In addition, the maximum entropy model only explains the bilateral action flow but cannot predict the change of input-output flow. The application of Markov models has brought the measurement of urban spatial interactions to the stage of statistical stochastic models. Markov models are currently more reasonable transfer models to describe probabilistic migration because of their non-sequential and ergodic nature (Turalska & West, 2014)

Intercity population flows are a reflection of all individual decisions and behaviours in the region, and at the same time reflect to some extent the evolution of intercity spatial interactions in the region. The mainstream spatial interaction models are all hypothetical models that cannot provide an accurate description of the population flow in an open system in an urban region. In order to better describe the spatial interactions within complex urban systems, Master Equation method can be introduced in Markov process. Master equation is often applied as a powerful tool in the study of open systems (Zhao et al., 2018). The use of master equation to represent the time evolution equation (differential form) of a Markov process enables a more reasonable representation of mobility probabilities and relates individual mobility behaviour at the micro level to changes in population dynamics at the macro level. Currently, the master equation has been introduced to model stochastic processes such as population dynamics. The core idea of the master equation is to first couple the density matrices of the physical environment and the open system, then calculate the overall density matrix evolution equation, and finally obtain the evolution equation of the open system through the physical environment bias traces (Li et al., 2021). The further partitioning of the state space by the relevant indicators during the calculation of the master equation can achieve higher accuracy of the solution. Therefore, the master equation can describe the time evolution of the joint probability distribution of all population flow trajectories in the open city system, which is a good simulation for the dynamically changing city system and the stochastic population flow, so as to obtain the spatial interaction forces that are more in line with the actual meaning. However, the current master equation study lacks quantitative index measures and evaluation mechanisms for inter-city interactions, and the ability to characterize and visually analyse spatial interactions is still weak.

In this paper, a spatial interaction model based on the master equation is constructed. The model introduces the master equation describing the stochastic flow process and realizes the modelling of spatial interactions of open urban systems with the characteristics of stochastic non-stationarity and dynamic variability. Using the Tencent migration big data of Beijing-Tianjin-Hebei region from 2016-2018 as the

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data source, the spatial interaction force between cities in Beijing-Tianjin-Hebei region is simulated using the master equation spatial interaction model, and the quantitative description and spatio-temporal characteristics analysis of the spatial interaction in Beijing-Tianjin-Hebei region are realized.

2.0 Study Area

The study area of this paper mainly includes Beijing, Tianjin, and 10 cities in Hebei, including Shijiazhuang, Tangshan, Cangzhou, Baoding, Qinhuangdao, Langfang, Chengde, Zhangjiakou, Xingtai, and Handan (See Figure 1). The population movement data in the experiment was derived from the Tencent Location Big Data platform (https://heat.qq.com/), which used location-based service technology LBS to collect population movement data from January 1, 2016, to December 31, 2018, a total of 1095 days and 341 cities. Data attributes include starting city, ending city, migration time, migration number, car ratio, train ratio, airplane ratio, starting city X, starting city Y, ending city X, ending city Y, distance and migration type. The urban population data are derived from the "year-end resident population" data of 341 cities in the 2016-2018 China Urban Statistical Yearbook as the base population data in the beginning of 2016-2018. With the development of remote sensing RS, GPS, location-based service technology LBS, location sharing service technology LSS and other technologies becoming more mature, the geospatial location, basic social attributes and mobile trajectory of users can be obtained in real time and comprehensively (Tabata et al., 2011). Compared with traditional survey data, geographic behaviour big data has the advantages of large sample size, good spatial-temporal completeness and strong analysis and prediction (Birch & Young, 2006). Demographic and economic indicators are obtained from the 2017-2019 China Urban Statistical Yearbook, with "year-end resident population" as the demographic indicator and "gross regional product" as the economic indicator, and the product of the two indicators is used to characterize city size. The distance between cities is obtained by calculating the spatial linear distance between the geometric canters of two cities.

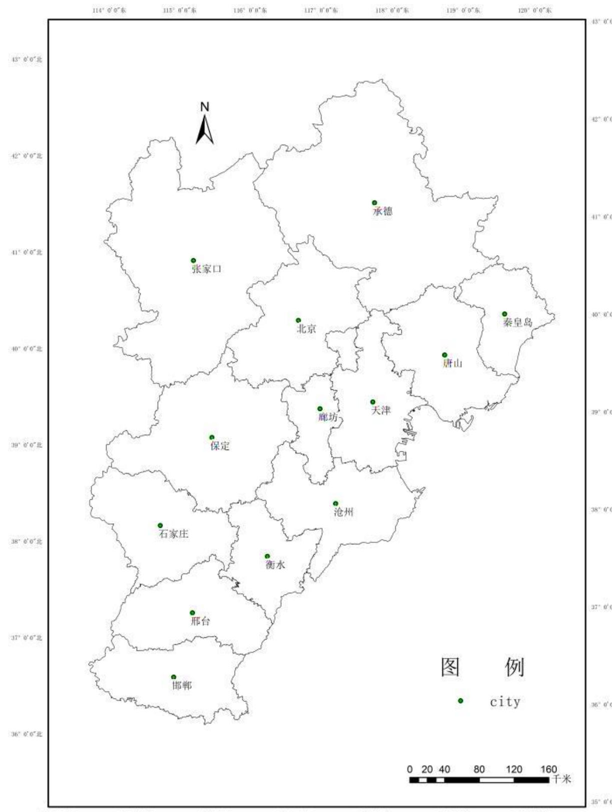

Figure 1: Study area.

3.0 Methods

3.1 Mathematical principle of the master equation

For complex evolutionary systems, the transfer of system states may exist in many different configurations, and the occurrence of different configurations may be synchronous and asynchronous in time, which usually makes it difficult to accurately obtain each precise state in the evolution of the system, but only partial information about the evolution of the system can be obtained at a specific time cross-section. Especially for open systems, the total number of elements within the system also changes continuously, making the modelling of its transfer probability more complicated. Traditional probabilistic models based on frequencies or Markov chains, etc., are difficult to provide an accurate description of the evolution probabilities of such complex systems. The master equation is used to describe the transformation and leap between different states of the system elements and is an equation of motion for the evolution of probability distribution over time. The master equation focuses on the inputs and outputs of the system, as well as the channels and patterns of interactions that may evolve in the system, and generally does not require assumptions about the properties of the system itself. Due to the openness of its state distribution and the dynamics of state transfer, the master equation can be applied to open systems as well as to non-equilibrium states (Stollenwerk & Briggs, 2000), and thus is often applied to many aspects of physics, chemistry, biology, etc (Ubaldi et al., 2016).

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Assume that the system is in a discrete state configuration, $\mathbf{n} = (n_1, n_2, \dots, n_L)$, the state n_L of different parts of the system constitutes the system state configuration $\mathbf{n}$. Over time, the state of each part of the system changes and the system state configuration shifts and changes, accordingly, described as the shift of the system state. The probability that the system may be in system state $\mathbf{n}$ at time t is defined as:

$$\sum_{\mathbf{n}} P(\mathbf{n}, t) = 1 \qquad [P(\mathbf{n}, t) \geq 0] \tag{1}$$

The system state transition probability is related to the conditional probability. If the system is in state $\mathbf{n}_1$ at time t_1, then the conditional probability of being in state $\mathbf{n}_2$ at time t_2 is:

$$p(\mathbf{n}_2, t_2 | \mathbf{n}_1, t_1) \geq 0 \tag{2}$$

Based on the Chapman-Kolmogorov equation (Haag, G. 2017), the basic form of master equation can be obtained:

$$\frac{dP(\mathbf{n}, t)}{dt} = \sum_{\mathbf{m}} w_t(\mathbf{n}, \mathbf{m}) P(\mathbf{m}, t) - \sum_{\mathbf{m}} w_t(\mathbf{m}, \mathbf{n}) P(\mathbf{n}, t) \tag{3}$$

Where: $\frac{dP(n,t)}{dt}$ denotes the amount of change in state n as probability at moment t; n is the current state; and m is the other states; $P(m, t)$ is the probability of being in the state m at moment t; $w_t(n, m)$ is the transfer rate of state m transition to state n. The master equation is a chi-square first order differential equation for the evolution of the system state configuration probability distribution over time, describing the change in the system state probability corresponding to the dynamic process of the system state transfer. The state transfer rate $w_t(n, m)$ is the mathematical meaning of the derivative of the state transfer probability, i.e., the derivative of the probability of a certain moment when other states m transition to the state n of the state transfer probability, which can be described as the magnitude of the change in the transfer probability over time. The master equation provides a detailed description of the evolution of the system under uncertainty or restricted information. In particular, the transfer rate in the master equation $w_t(\mathbf{n}, \mathbf{m})$ is changing gradually with time. In Equation 3, the $w_t(\mathbf{n}, \mathbf{m})$ represents the rate of transfer between states for each time unit and $w_t(\mathbf{n}, \mathbf{m})P(\mathbf{m}, t)$ is the rate at which each time unit is transferred from state $\mathbf{m}$ to state $\mathbf{n}$, The probability of change of state distribution $\frac{dP(\mathbf{n},t)}{dt}$ (state $\mathbf{n}$ probability change per unit time) is the result of the offset of two opposite effects on the right side of the equal sign. The probability $P(\mathbf{n}, t)$ of the unit time change is caused by these probability flux differences, which are obtained by the conversion of other different states. Unlike the Markov chain where the next state depends only on the previous state, the change in the state distribution of the master equation is obtained through the transformation of other states and can adequately describe the openness of the change in the state distribution

3.2 Master equation population mobility model

For a system of cities in a region, we can use the state vectors n=(n_1,n_2,...n_L) to describe the population of each city and satisfies:

$$\sum_{i=1}^{L} n_i = N \tag{4}$$

Where N is the total population of the urban system, and n_i is the number of populations in the city i。The number of populations mentioned in this paper refers to the resident population unless otherwise indicated.

Further, for the changing population distribution in an urban system, the master equation can be formulated as follows Derivation:

$$\frac{dP(\mathbf{n}, t)}{dt} = \sum_{\mathbf{m}} w_t(\mathbf{n}, \mathbf{m}) P(\mathbf{m}, t) - \sum_{m} w_t(\mathbf{m}, \mathbf{n}) P(\mathbf{n}, t) \tag{5}$$

Where n is the current state of population distribution in the urban system; mis the state of population distribution different from n； P(m,t) is the probability that the population distribution state of the urban system is n at time t； w_t (n,m) is the state of population distribution of the urban system at time t from n to m the transfer rate.

The master equation describes the dynamic process of the evolution of the state of the population distribution of an urban system over time. It is not intuitive to use the probability change of the state to represent the dynamics of the huge number of populations in the actual urban system. A more scientific approach is to use the product of the probability of the state and the component to calculate the average value of the number of urban populations corresponding to that component. Based on the master equation formula (5) and the formula for calculating the mean of discrete quantities, the following formula is obtained:

$$\frac{d}{dt}\langle n_i \rangle_t = \sum_{n} n_i \frac{dP(\mathbf{n}, t)}{dt}$$
$$= \sum_{\mathbf{n}} n_i \left(\sum_{m} w_t(\mathbf{n}, \mathbf{m}) P(\mathbf{m}, t) - \sum_{m} w_t(\mathbf{m}, n) P(\mathbf{n}, t) \right) \tag{6}$$

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where: n_i is the population of the city i when the population distribution of the urban system is in state and ⟨n_i ⟩_t is the average population of the city at time t.

The state transfer ratew_t whose mathematical structure is determined by the system under consideration, determines the complexity of the master equation and is the focus of modelling efforts. In practical applications, the total transfer rate can be decomposed into the transfer rate of the internal and external system actions. The state change of population distribution in an urban system originates from the population movement between cities and the natural growth of population, so the state transfer rate isw_t is refined into three components: urban population mobility, birth rate, and mortality rate:

$$w_t(\mathbf{n}+\mathbf{k},\mathbf{n}) = \sum_{i,j} w_{ji}(\mathbf{n}+\mathbf{k},\mathbf{n}) + \sum_{i} w_{i+}(\mathbf{n}+\mathbf{k},\mathbf{n}) + \sum_{i} w_{i-}(\mathbf{n}+\mathbf{k},\mathbf{n}) \quad (7)$$

$$\mathbf{k} = (0,\ldots,+1_i,\ldots-1_j,\ldots,0)$$

Introducing the shift operator E^(±1) (Haag, G. 2017), the above equation is processed to finally obtain the exact equation of motion for the mean urban population size at moment t, i.e., the main equation population flow model:

$$\frac{d\langle n_j\rangle}{dt} = \sum_{i=1}^{L}\Big(w_{ij}(\mathbf{n}) - w_{ji}(\mathbf{n})\Big) + \langle w_{j+}\rangle - \langle w_{j-}\rangle \quad (8)$$

Where:d/dt ⟨n_j ⟩ is the city j population change;w_ij (n) is the urban j to the city i the number of population movements.⟨w_(j+) ⟩-⟨w_(j-) ⟩is the city j the natural growth rate itself (the effect of natural growth is ignored in the population movement experiment).

3.3 Master equation spatial interaction model

Ignoring natural population growth, the dynamics of population distribution depends only on the movement of people between citiesw_ij. The mobility process is influenced not only by socioeconomic and other social indicators, but also by the interaction between different members of society. In turn, population mobility causes changes not only in the overall social indicators, but also in the attitudes and behaviour of other members of society (Haag, 2017; Zhang, 2016). Population mobility can be modelled as follows:

$$w_{ij} = n_j p_{ij}(\mathbf{n}) \quad (9)$$

Where w_ij is the flow from the city j flow to the city i; n_j is the urbanj population; p_ij is the probability that the population of the city j moves to the city i per unit time.

Population mobility rate p_ij i.e., the number of people moving from urban-to-urban areas per unit of time j of the population moving to the city i of the city, which we use to summarize the probability that j all individuals to move to the city i. In order to ensure that the population movement between cities is in line with objective laws and that the number of population movements is positive, the mobility rate p_ij must be a positive number, and to ensure that generality is not lost, an exponential function is used for conversion and representation. the Weber-Fechner law describes people's quantitative responses to external physical stimuli, and according to Fechner's findings and mathematical formalization, subjective perceptions are proportional to the logarithm of the stimulus intensity, and the inverse effect is also used as an exponential correlation. From this, it can be determined that the relationship between the rate of transfer of individuals and the city is a positive exponential function, obtaining the following expression for the rate of population mobility.

$$p_{ij}(\mathbf{n}) = v_{ij}\exp\Big(u_i(n_i+1) - u_j(n_j)\Big) \geq 0 \quad (10)$$

The urban spatial interaction model based on the master equation is obtained through Equations 9 and 10:

$$w_{ij}(\mathbf{n}) = \langle n_i\rangle p_{ij}(\mathbf{n}) = \langle n_i\rangle v_{ij}\exp\Big(u_i(n_i+1) - u_j(n_j)\Big) \geq 0 \quad (11)$$

Where p_ij (n) is the population mobility rate.v_ij is the interaction coefficient, describing the city i with the city j the frequency of interaction.u_i is the dynamic utility of the city i.

3.3 Spatial Interaction Measure - Interaction Coefficient

Interaction Coefficient, v describes the frequency of interaction between cities within an urban system and reflects the degree of cooperation and population exchange between two cities, but does not reflect the strength of the city's attractiveness. In thev matrix, a single interaction coefficient v_ij contains influences that facilitate or hinder the shift in population movement between cities, the interaction coefficients can be obtained using the nonlinear estimating equation:

$$v_{ij} = \frac{n_i w_{ji}\exp(u_j - u_i) + n_i w_{ij}\exp(u_j - u_i)}{n_i^2\exp 2(u_j - u_i) + n_j^2\exp 2(u_i - u_j)} \quad (12)$$

Where n_i is the population data for city i; w_ij is the population flow between city i and city j; u_i and u_j are the city i with the city j the urban utility of the city.

4.0 Results

4.1 Feature importance

The spatial interactions between cities were calculated separately using the master equation spatial interaction model for 2016-2018, and the results were presented in the form of a network linkage map (Figure 2).

The results show that Beijing's spatial interaction with other cities is particularly significant, fully reflecting the central position Beijing occupies in the Beijing-Tianjin-Hebei region. Shijiazhuang, Langfang, Tianjin and Baoding are in the second tier, and all four cities occupy a good geographical location - in the central part of the Beijing-Tianjin-Hebei region and close to Beijing - and have a strong ability to connect with other cities while being subject to Beijing's high-intensity spatial interaction force. Langfang jumped to second place in 2018, a year in which its ties with the two major municipalities of Beijing and Tianjin were greatly strengthened. In addition, there are also strong spatial interactions between Tangshan-Qinhuangdao, Handan-Xingtai, and Cangzhou-Hengshui, and all three groups of cities are geographically adjacent to each other, with stronger circulation and collaboration compared to other cities in terms of population flow, industrial composition, and information exchange.

The analysis shows that the spatial network of Beijing-Tianjin-Hebei region shows an integrated development network pattern with Beijing as the core radial network and the network of small central cities as a supplement. Some areas have achieved a more ideal "core-sub-centre-periphery" network structure. As a polycentric city, Beijing is rich in political, economic, technological and cultural resources, and is able to form strong interactions with neighbouring cities. In terms of industrial collaboration, Beijing has formed distinctive industrial clusters with cities such as Baoding, Tianjin, Langfang and Zhangjiakou. The southern region of Beijing-Tianjin-Hebei, with Shijiazhuang as the sub-centre, also forms a higher intensity connection. Shijiazhuang is on the same major transportation route as Handan and Xingtai, and the convenient transportation is one of the factors that prompt the three to form a mini network. The proposed Shi-Han intercity railroad plan will further deepen the connection of this mini-group network in the south and expand the population attraction.

The emergence of a small network of sub-canters in the Beijing-Tianjin-Hebei region indicates that the center of gravity in the region is being dispersed outward from Beijing, which is conducive to relieving the pressure on the Beijing region, driving the development of peripheral "thin" cities, and eventually forming an integrated development network for the entire Beijing-Tianjin-Hebei region. However, at the present stage, some cities have the problems of small-scale industries and slow development, and the intensity of connection with other cities is low, and the development differences between cities are large. Promoting regional integrated development and reducing regional development differences are issues to be solved for the further development of the Beijing-Tianjin-Hebei region.

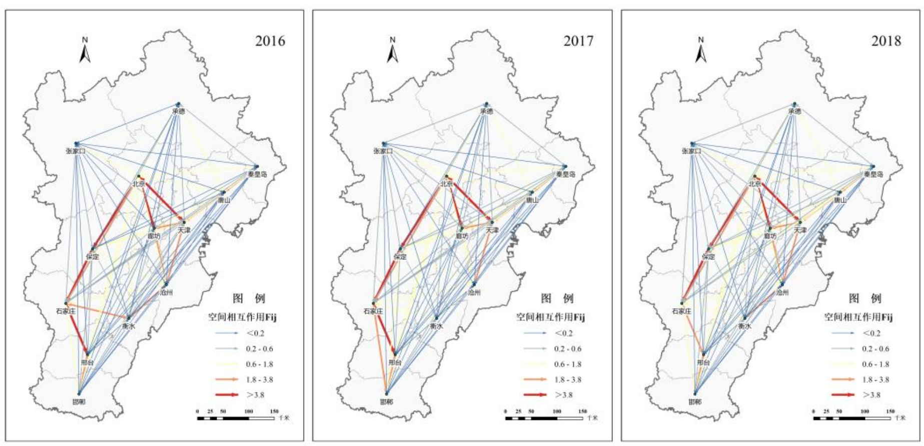

Figure 2: Spatial interconnectedness intensity map of Beijing-Tianjin-Hebei region from 2016- 2018.

In order to show the excellent simulation ability of the master equation spatial interaction model for urban interactions, this paper refers to the classical gravity model (Equation 13) and uses relevant data from the 2016-2018 China Urban Statistical Yearbook to calculate the urban interactions in the Beijing-Tianjin-Hebei region, and the results are shown in Figure 3.

$$F'_{ij} = a \times \frac{Q_i \cdot Q_j}{{d_{ij}}^{\beta}} \tag{13}$$

One, based on the classical gravitational model shows that the cities with stronger interactions are mainly concentrated in the Beijing-ring city circle entered on Beijing and Tianjin, including Baoding, Tangshan, Langfang, Shijiazhuang and Cangzhou, showing a strong grouping, and the influence of the rest of the cities is not significant; and only Shijiazhuang in the south shows a strong spatial interaction with Baoding, and does not show the southern city with Shijiazhuang as the sub-centre The spatial interconnectedness of the southern urban circle with Shijiazhuang as the sub-centre is not demonstrated. In contrast, the main equation spatial interaction model simulates a circle-like development network structure, which can show more details of the overall urban spatial interaction pattern in the Beijing-Tianjin-Hebei region. Secondly, due to the limitation and lag of the yearbook data, the interaction intensity of some Beijing-Tianjin-Hebei cities located in Hebei province basically showed a change trend of decreasing before increasing in 2016-2018. Affected by the policy of emission reduction and energy conservation and industrial restructuring, the regional GDP of Hebei Province in 2017 showed negative growth compared with 2016, and the shrinkage was about three times. Among them, the total GDP value of Chengde City in 2018 is still negative growth compared with 2016. The classical gravity model treats cities as hypothetical closed cities, and the total GDP value is one of its important indicators for calculating city size, which is positively correlated with the intensity of city interactions. While the master equation spatial interaction model emphasizes the stochastic and open nature of the urban system, this study takes the perspective of population mobility to invert the evolution of urban spatial interaction in an open system.

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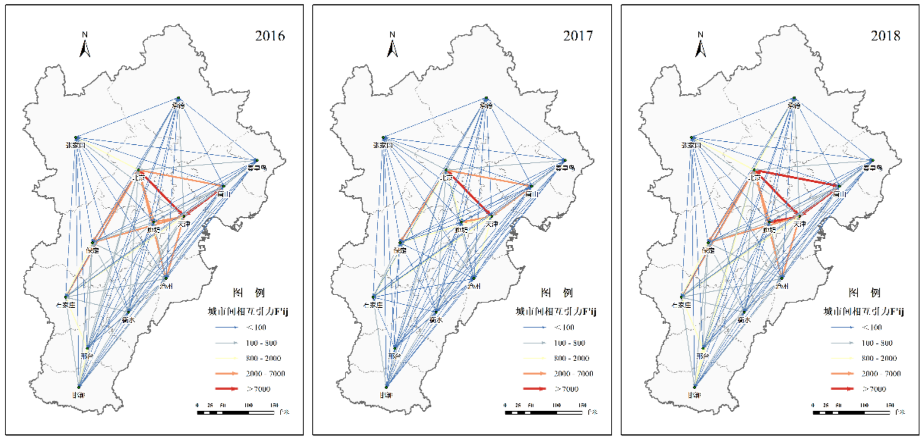

Figure 3: Map of mutual gravitational strength between cities in Beijing-Tianjin-Hebei region based on classical gravitational mode.

6.0 Discussion

The traditional deterministic modelling stage of spatial interactions is based on closed deterministic urban models, which do not sufficiently characterize the stochastic and dynamic nature of realistic open urban systems. This paper constructs a master equation spatial interaction model based on the master equation method, , to describe the dynamics and stochastic of the urban development process by introducing parameters such as urban mobile population , so as to simulate the urban spatial interaction in the open system and calculate the related measures, and then discover the spatial pattern of urban and inter-city development based on the big data of population mobility, which is important for guiding the future sustainable development of cities and regions. It is of great practical significance to guide the sustainable development of cities and regions in the future. Specifically, the master equation spatial interaction model has the following characteristics.

Dynamicity. The dynamics of urban systems are described by observing inter-city population flows, population flows are the spatial reallocation process of production factors, and in urban networks, population flows indirectly reflect inter-city economic exchanges with accompanying social and economic values. Compared with the traditional spatial interaction model, the master equation spatial interaction model can better describe open the stochastic and dynamic relationships of urban systems, and has a greater advantage in statistics and analysis of discrete values.

Openness. Population mobility takes place throughout urban networks, with network city nodes connecting and exchanging with multiple cities. In urban networks, individual cities have limited mobility in their own social stratification, and the mobility of social members is often achieved through geospatial mobility. The master equation spatial interaction model proposed in this paper instead uses the big data of population mobility between cities and multiple cities as the base data, regards the change of population mobility over time as the premise, and constructs the master equation with urban spatial interaction as the target, so the master equation spatial interaction model is able to calculate the openness presented in the selected area in the openness system.

6.0 Conclusions

In this paper, the intercity spatial interaction was calculated by the master equation interaction model in Beijing-Tianjin-Hebei region and compared with the simulation results of classical gravity model, and the findings are as follows.

(1) The calculation of spatial interaction in the Beijing-Tianjin-Hebei region from 2016 to 2018 reveals that Beijing, as the core city of the synergistic development of the Beijing-Tianjin-Hebei region, is in the core position of the regional urban system. Tianjin, Shijiazhuang, Baoding and Langfang, as the four cities with the largest combined scale in the Beijing-Tianjin-Hebei region, are second only to Beijing in terms of spatial interaction intensity. Tianjin and Shijiazhuang form sub-centre clusters in the east and south of the Beijing-Tianjin-Hebei region, respectively, forming industrial collaboration and resource sharing with Tangshan, Xingtai, Handan and other cities, and driving the development of some peripheral cities.

(2) The main equation spatial interaction model can show the strength of inter-city interaction in more detail than the classical gravity model. The urban interaction calculated by the classical gravity model mainly focuses on the Beijing-Tianjin city ring with Beijing-Tianjin as the twin canters, and the gradation of each echelon of cities is not significant and the results are bi-polarized. The spatial interaction model simulation results of the master equation show that the spatial interaction in Beijing-Tianjin-Hebei region shows a circle network development pattern of "one core and many canters", highlighting the southern city network development structure with Shijiazhuang as the centre.

Due to the availability of population flow data, the data used in this paper are consecutive years from 2016 to 2018, with a small span of years, which has limitations in reflecting the evolution pattern of spatial interaction of population flow over a longer time scale; in addition, the spatial interaction model constructed based on the master equation method used in this paper needs further improvement in the acquisition of relevant variables and parameters. In addition, the spatial interaction model based on the master equation approach needs to be further improved in terms of variables and parameters to obtain more realistic findings and make the policy implications related to the synergistic development of Beijing-Tianjin-Hebei region more relevant.

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References

Birch, D. A., & Young, W. R. (2006). A master equation for a spatial population model with pair interactions. *Theor Popul Biol, 70*(1), 26-42. doi:10.1016/j.tpb.2005.11.007

Haag, G. (2017). Modelling with the Master equation. New York, NY: Springer International Publishing.

Jianhua, H., Xuan, S., Jian, G. J. G., & University, I. S. o. W. (2016). Modeling the spatial expansion of urban agglomeration considering their spatial interaction: a case study of Wuhan Metropolitan Area. *Journal of Geomatics and Information Science of Wuhan University, 41*(4), 462-467.

Lee, K., Goh, S., Park, J. S., Jung, W.-S., & Choi, M. Y. (2011). Master equation approach to the intra-urban passenger flow and application to the Metropolitan Seoul Subway system. *Journal of Physics A: Mathematical and Theoretical, 44*(11). doi:10.1088/1751-8113/44/11/115007

Xiaoyu, L., Yong, H., Junyi, L., & Qinsheng, Z. (2021). Hidden Markov Model Based on the Quantum Conditional Master Equation. *Journal of University of Electronic Science and Technology of China, 50*(5): 644-649. doi: 10.12178/1001-0548.2021241

Li, S., Wang, Z., & Zhong, Z. (2012). Gravity model for tourism spatial interaction: Basic form, parameter estimation, and applications. *J. Acta Geographica Sinica, 67*(4), 526-544.

Olsson, G. (1967). Central place systems, spatial interaction, and stochastic processes. Paper presented at the Papers of the Regional Science Association.

Sobkowicz, P., Thelwall, M., Buckley, K., Paltoglou, G., & Sobkowicz, A. (2013). Lognormal distributions of user post lengths in Internet discussions-a consequence of the Weber-Fechner law?, *EPJ Data Sci., 2*, 1-20.

Stollenwerk, N., & Briggs, K. (2000). Master equation solution of a plant disease model. *Physics Letter A, 274*(1-2), 84-91.

Tabata, M., Eshima, N., & Takagi, I. (2011). A mathematical modeling approach to the formation of urban and rural areas: Convergence of global solutions of the mixed problem for the master equation in sociodynamics. *Nonlinear Analysis: Real World Applications, 12*(6), 3261-3293. doi:10.1016/j.nonrwa.2011.05.025

Turalska, M., & West, B. J. (2014). Critical social networks. *Physica A: Statistical Mechanics and its Applications, 395*, 466-475. doi:10.1016/j.physa.2013.10.033

Ubaldi, E., Perra, N., Karsai, M., Vezzani, A., Burioni, R., & Vespignani, A. (2016). Asymptotic theory of time-varying social networks with heterogeneous activity and tie allocation. *Sci Rep, 6*, 35724. doi:10.1038/srep35724

Wang Shanyan, Wang Shijun, Tian Junfeng. (2015). Comparative research about Ha-Chang and Mid-Southern Liaoning urban agglomerations based on the intensity of urban flow. *Economic Geography, 35*(11): 94-100, 116. doi: 10.15957/j.cnki.jjdl.2015.11.013

Zhao, Z., Wang, J., & Zhao, J. (2018). A spatial analysis of urban economic connections among the node cities along the "One Belt and One Road" in China. 32, 12-18.